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Relationship of Preferred Tone Reproduction to Picture Size

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RELATIONSHIP OF PREFERRED
TONE REPRODUCTION TO PICTURE SIZE

by David C. Miller

A thesis submitted in partial fulfillment of the requirements for the degree Bachelor of Science in the School of Photography in the College of Graphic Arts and Photography of the Rochester Institute of Technology.

January 1973

Thesis Adviser: Dr. J.A.C. Yule

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In particular my appreciation goes to Dr. John A. C. Yule for his valuable advice and patient teachings. The value of these will continue long after the paper is complete.

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From Kodak, J. L. Simonds and, especially, C. N. Nelson contributed important insight into the project basics.

In addition my wife, Gail, deserves much credit for her patience in the many hours devoted to this project and especially for her much needed assistance in the typing of this work.

Abstract

A relationship between preferred tone reproduction and final picture size is suggested in many forms in the literature and in practical experience.

This project uses photographic halftone prints varying in tone reproduction and produced at different sizes to investigate this relationship. The tone reproduction variations are introduced electronically using an RCA color scanner and a method of eigenvector analysis is used to reduce these tone reproductions down to their important components.

The final set of prints consisting of ten tone reproductions, three reproduced sizes, and two sharpnesses are evaluated to determine their subjective quality in the judgement of a group of ten observers.

These quality ratings are normalized and averaged and then mathematically correlated with an eigenvector representation of their tone reproduction. The preferred tone reproduction (producing maximum quality) may then be determined for each picture size.

Due to large differences in quality rating from observer to observer and an incomplete selection of tone reproductions, no definite relationship was determined. Useful information was obtained on the use of the eigenvector method, the magnitude of quality variation as tone reproduction is changed, the difficulties of subjective evaluation, and a calculated tone reproduction producing maximum quality for a normal print not considering size.

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Section One

Introduction

The search for a method of controlling the reproduction process goes back many years. In 1890 Hurter and Drifffield stated, "The production of a perfect picture by means of photography is an art; the production of a technically perfect negative is a science." They went on to explain the photographic process in terms of log exposure input producing a log opacity (optical density) output characterized by the familiar D log E curve.¹ With this tool photography could move toward becoming a quantitative science.

It was not until L. A. Jones described his process of "tone reproduction" that an objective could be assigned to the photographic system. This objective he stated as exact reproduction of relative luminance. Many other studies have been made to determine, under controlled viewing conditions, what sort of objective tone reproduction characteristics are required to produce photographs of high quality as judged by many observers.^{2a,3,4}

In one such study⁵ the objective tone reproduction curves shown below were described. The important points to grasp are first, that scenes with different luminance ranges must be reproduced differently and second, the curves lie below the line representing 1:1 relative luminance reproduction.

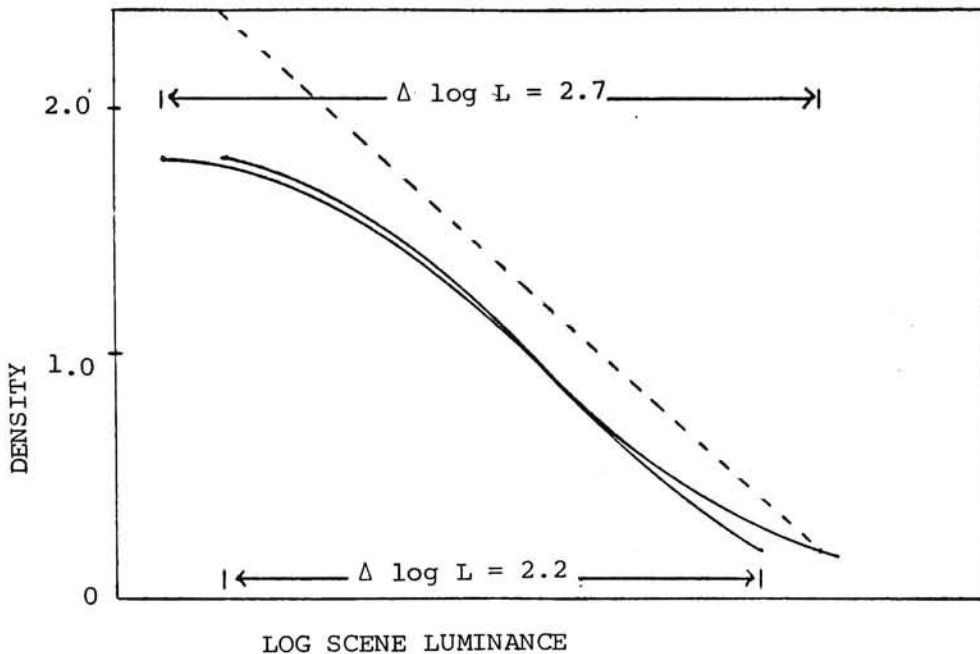


FIGURE 1A

Preferred Objective Tone Reproduction Curves

Another approach to the realization of optimum tone reproduction has been the more recent study of subjective tone reproduction. This study is involved in understanding the visual sensation (brightness) produced by a scene stimuli (luminance). This is a complicated procedure because the visual system is constantly changing adaptation so that there is seldom a constant relationship between brightness and luminance.^{2a} In studies where adaptation curves have been determined and subjective tone reproduction curves produced, it has been found that a 1:1 reproduction of original brightness vs. reproduction brightness, (relative to white), will produce the best subjective quality evaluations.^{6,7}

These studies are important because they give insight into and draw attention to the processes of visual adaptation. The evaluation of any process involving the human visual system can be complicated by

the fact that what the eye sees is not necessarily what the brain perceives. One such complication is the phenomena of simultaneous contrast which refers to the brightness of the surround and its effect on the appearance of an object.⁸ A gray patch appears lighter when surrounded by a dark area than it does when surrounded with a light area. This effect varies with the angular subtense of the areas involved so that it might not be visible in a large area outdoors but would be very apparent if the scene were reduced down to an 8 x 10 print.¹

The phenomena of successive contrast occurs when the eye fixates on different areas of a scene. The previous fixation has an effect on the adaptation of the eye for the next fixation and this occurs throughout the viewing of any object.¹

There would seem to be reason to believe that the angular subtense of an object on the retina would cause variations in contrast. This would be caused by the change in sensitivity of the light receptors as the image moves away from the center of the image area. Also the analysis of information content which varies fixation patterns may have some effect on adaptation.^{9,10,11}

Large area reproduction is not the only factor involved in quality rating of photographs. Small area image-structure characteristics are very important in the overall subjective evaluation of a picture. Graininess, sharpness, and resolving power contribute to the quality and definition ordinarily used to describe print preference.^{12,13,14,2b}

Thus we see that quality of a print depends on many interacting and widely variable aspects. Large scale tone reproduction

and complex visual phenomena, as well as small scale image-structure characteristics, all act together to influence a subjective opinion of quality.

When reproductions are changed in size many of these factors will be affected. Edge sharpness increases and fine-detail recognition decreases as reproductions are made smaller. Viewing conditions will change and at the same time it may be expected that some of the mechanisms of visual perception will be modified. It should be expected, then, that the subjective opinions of quality will also change.

It is the object of this project that a relationship be found between these changes in picture size and preferred tone reproduction. With such information the production of a perfect picture will move another step closer to becoming a science and away from the art.

Section Two

Outline

- 1) selection of appropriate methods of production and analysis
- 2) creation of an eigenvector computer program and determination of its suitability for use with the scanner gradation range
- 3) standardization procedures leading to the production of the halftone photographic reproductions
- 4) production of scanner positives, halftones, and prints for a preliminary experiment
- 5) evaluation of prints for quality
- 6) production of scanner positives, varying sizes of halftones and prints for final experiment
- 7) analysis of curve shapes with eigenvector program
- 8) production of standard viewing background for evaluations
- 9) evaluation by a group of observers
- 10) analysis of ratings and conclusions about relationship of preferred tone reproduction to final print size

Section Three

Procedures

The objective of this research project will be to determine the relationship between objective tone reproduction and subjective print quality as a function of picture size. From such a relationship the preferred tone reproduction corresponding to optimum print quality may be determined for each picture size.

In reaching such an objective, procedures for changing tone reproduction and picture size must be established as well as a method of quality determination. When this has been completed, the variables may be correlated mathematically to obtain the final relationship.

A. Tone Reproduction Variation

Several types of tone reproduction are usually considered subjectively significant. First, the choice of luminances in the original subject to be reproduced as highlight and shadow; second, the density range with which these highlight and shadow points are reproduced; and third, the variation in contrast in the highlight, middletone, and shadow areas of the reproduction.

Using the techniques of graphic arts reproduction our choices are reduced to a workable number. Ordinarily a photographic print of the original scene is used and in such a case the highlight and shadow luminances have been condensed into the density range of the print. In the printed reproduction the density range of the picture is limited by the paper printed upon and the density of the ink

layer applied. While either of these limitations may be modified somewhat, they will be considered as constant factors. We are then left with modification in curve shape as our remaining tone reproduction control. In graphic arts photography this control is exercised by producing halftones with varying amounts of main, flash, and bump exposure.*

In this project, however, a method of tone reproduction control with greater flexibility than this would be desirable. One method which provides this flexibility and which is available for research is the RCA (Hell) Chromograph C286 color scanner at the Graphic Arts Research Center. This instrument allows control of tone reproduction continuously, over a very wide range, by electronic means. The highlight, middletone, and shadow regions are individually adjustable and the maximum and minimum densities at the highlight and shadow may be kept constant.

The RCA scanner is ordinarily used to produce continuous tone separations from a color transparency or print utilizing electronic color correction and gradation controls. The procedure it uses is to pass a light beam through the original, which is spinning on a drum, and then through color filters and onto photomultiplier tubes where it is converted to an electrical signal. This signal travels to a computer section where it is modified for color correction, gradation, unsharp

*Main - image exposure through a halftone screen.
 Bump - additional image exposure without a screen
 (increases highlight contrast).
 Flash - uniform fogging exposure through screen
 (lowers shadow contrast and decreases density range of the negative).

masking and other procedures useful in the graphic arts. After modification this signal travels to the exposing section where it is reconverted to a light beam which exposes a sheet of film rotating in synchronization with the original, producing either a positive or a negative of the original at the operators option. The original chosen was a black and white print (for its defined highlight and shadow). This was combined with alphanumeric targets and density scales to produce the original as shown in Figure 3A. The output from the scanner will be a film positive.



FIGURE 3A
Format of Original Subject

The scanner adjustments were explored and the best procedures for adjusting the tone reproduction were determined. This is covered in much more detail in Appendix A.

Next the range of tone reproduction adjustment was determined. The procedure was to adjust each gradation control from maximum to minimum leaving the endpoint densities constant. Each gradation control

was set to each of three positions to produce a total of twenty-seven different tone reproduction curves.

A portion of this set of curves is shown below in Figure 3B with the density of the original plotted against voltage to the scanner exposure lamp (proportional to output density). The range of tone reproduction available is very large and should be entirely sufficient for this project.

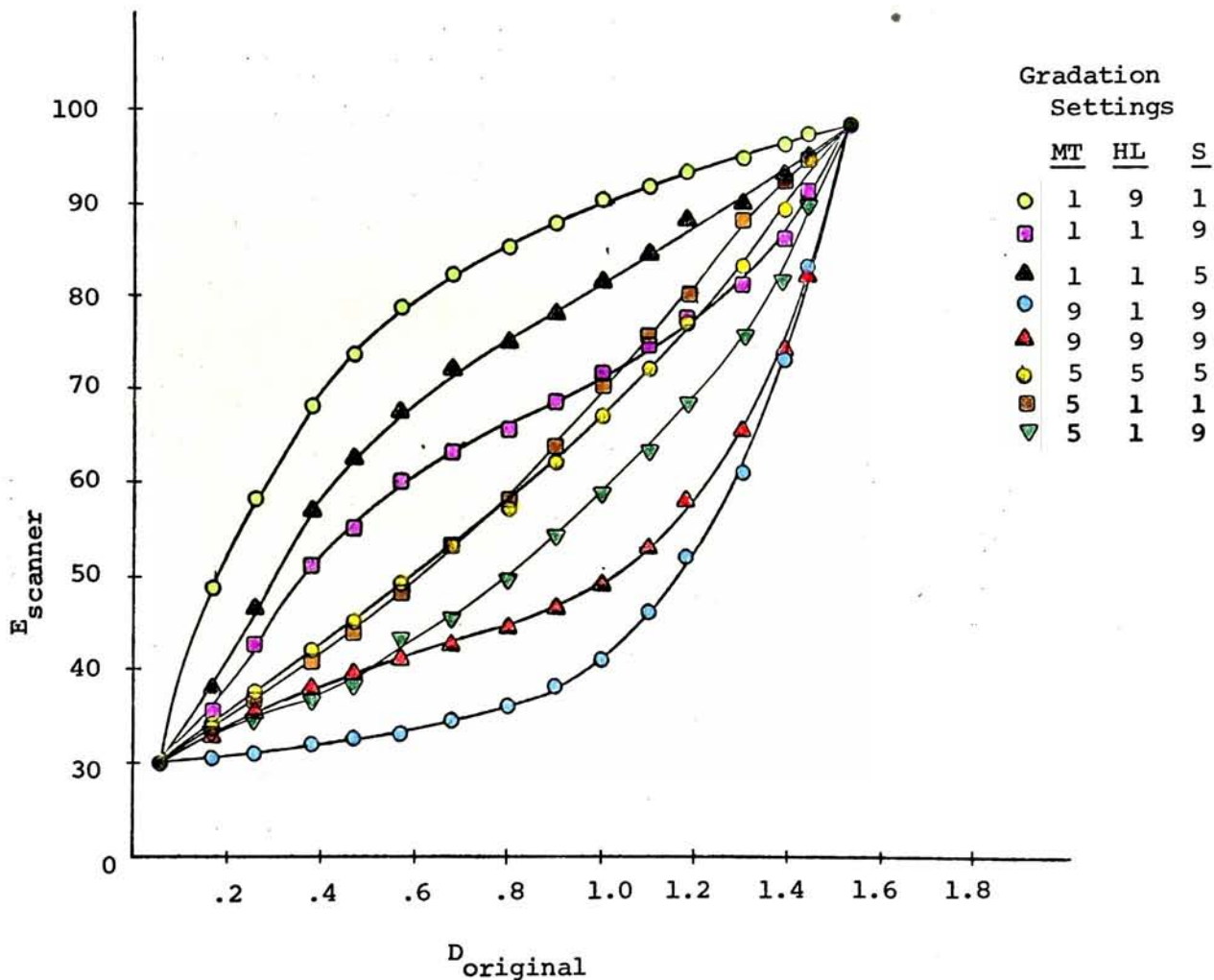


FIGURE 3B - Scanner Gradation Range
(plotted on non-linear Munsell scale)

B. Eigenvector Analysis

In order to perform the necessary mathematical analysis at the end of the project, the tone reproduction curves should be reduced to a minimum number of descriptive parameters. One method of achieving this is eigenvector analysis. This method (used in multivariate analysis) is used to obtain vectors which when added in various linear combinations will recreate the original set of data (usually represented by a set of curves) utilizing far fewer parameters.

These vectors are ordinarily called eigenvectors (V) and the amount of each vector needed to recreate the original data is called the scalar multiple (λ). For a given set of data the mean vector, eigenvectors, and scalar multiples may be found which can recreate the original data points as follows:

$$z = \bar{z} + v_1 \lambda_1 + v_2 \lambda_2 + v_3 \lambda_3 + \dots$$

Since the mean vector and eigenvectors are the same for any curve from the original set, any curve may be described by specifying its scalar multiples. There is a further description of this method in Appendix B.^{16,18,19,20}

A computer program was written which would perform the necessary calculations and this was applied to the family of twenty-seven curves produced by the scanner. When this was done each curve could be reduced to a specification of only three scalar multiples. Thus instead of the sixteen steps originally used to describe each curve, each one could now be described using three numbers. This indicates that the eigenvector method should be useful in the necessary

mathematical analysis and this method is applicable to the family of gradation curves produced by the RCA scanner.

C. Halftone and Print Production

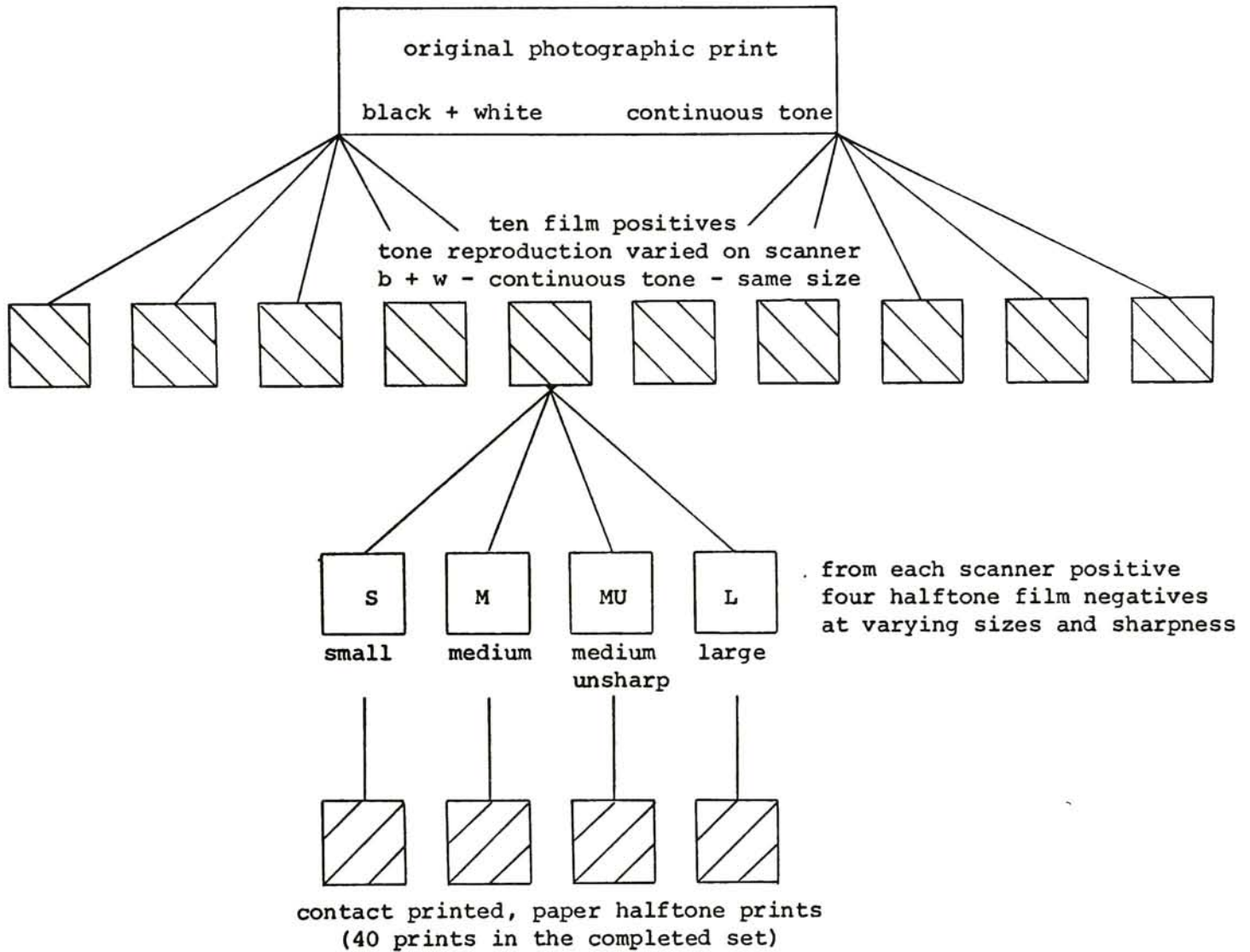
Before anything may be produced from the scanner, the remainder of the system must be standardized. The necessary requirement is that the density produced on the final photographic halftone print be predictable as one goes through the halftone and printing stages. Analysis with a system such as the Jones diagram is useful for cascading a number of operations as is done in this project.

In the production of the halftone negatives, a Kodak 150 L magenta positive screen was used and the Kodalith Ortho Type 3 films were processed in a Log E processor using RT developer and replenisher. A Klimsch graphic arts camera was used for each halftone exposure with the scanner positive back-lighted in the transparency holder. Each halftone was produced using a main and bump exposure. The purpose of this dual exposure was to allow some compensation for variations in the density range of different scanner positives and to adjust for small changes in the development activity of the automatic processor. This compensation was performed by changing slightly the ratio of main to bump exposure which will vary the basic density range (BDR)* of the halftone screen.

An investigation of sharpness is significant in a study of this nature. It is felt by J. L. Simonds of Kodak (private communication)

*BDR - the range of densities on the original which will produce the chosen highlight and shadow dot size.

Figure 3



FLOW CHART OF PRODUCTION PROCEDURE

that sharpness is the only important source of changes in preferred tone reproduction as size changes. This may be tested by producing an unsharp picture at the halftone stage and including this with those to be evaluated.

The print production was done by contact printing each halftone onto Kodabromide E-2 paper using a point light source and the vacuum film back of the Klimsch camera.

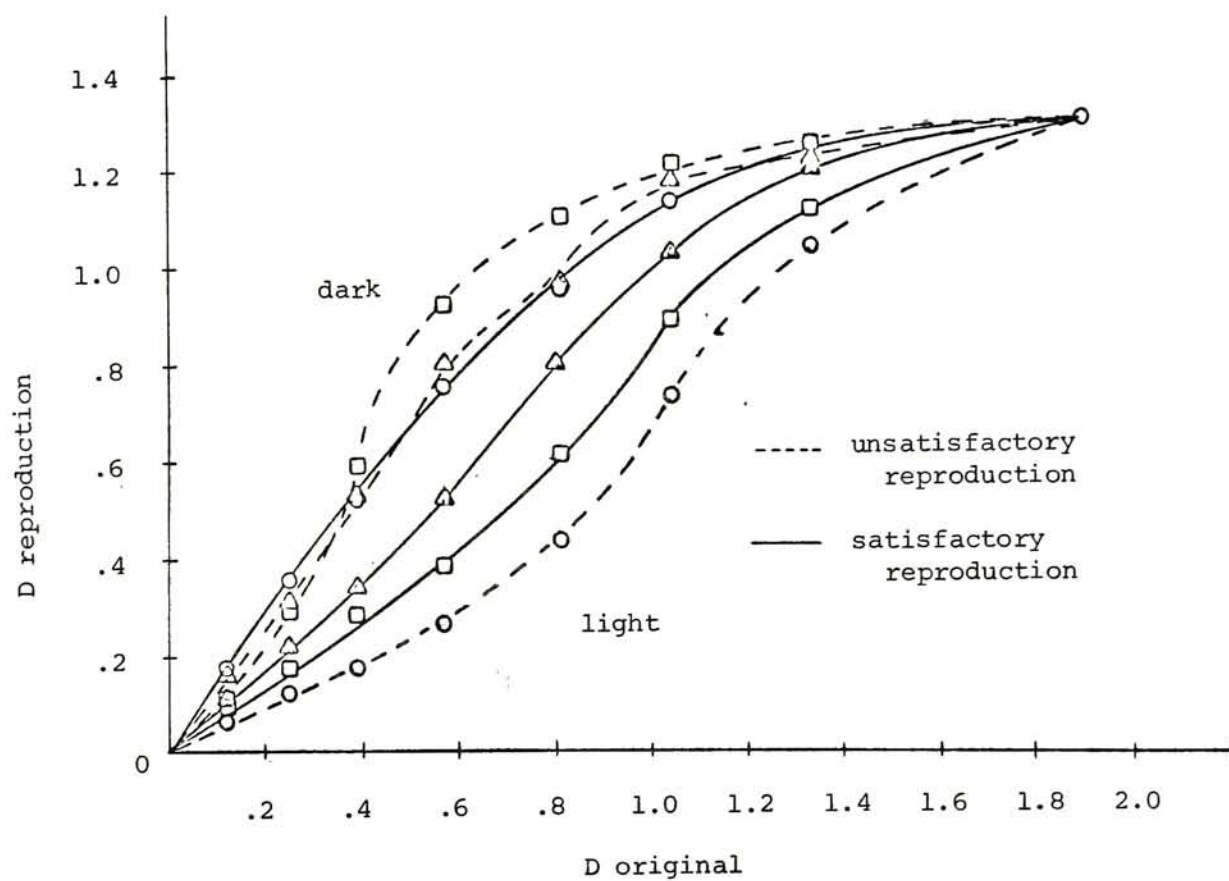
Kodabromide E-2 paper was chosen for its surface qualities and gloss which approximate ink on a coated paper stock. This paper is exposed and processed to produce a maximum density of 1.30 which is typical of monochrome printing on a lithographic press.³ Each print is tray processed in Dektol developer. The details on processing and set-up for both the halftone and printing stages are given in much more detail in Appendix A.

D. Preliminary Experiment

Using the procedure established in previous sections, a preliminary experiment was performed. This preliminary experiment was a scaled down version of the final print production and was designed to check the procedures which were laid out and, most important, to determine the range of tone reproductions which should be covered in the final experiment. Eight prints were produced which varied in tone reproduction. Three observers rated each print as acceptable or not acceptable in overall quality. Such a procedure gives very little information about preferred tone reproduction but instead indicates, as shown below, an approximate range in which the experiment should be made.

FIGURE 3C

Results of Preliminary Experiment



E. Final Experiment

Using the set of curves from the preliminary experiment, a range of tone reproductions within the acceptable region is chosen for study. Using the RCA scanner, twenty film positives were made from the original print, each with a different tone reproduction. These were then compared with those known to be acceptable and ten were chosen as being representative of the range to be tested.

From the ten scanner positives, halftones were made as described previously. From each of the scanner positives, five halftones were made. Three of them varying in magnification and two varying in size and sharpness. This is shown in Figure 3D below with the magnification from the original (magnification ranges vary by 2 1/2 times)

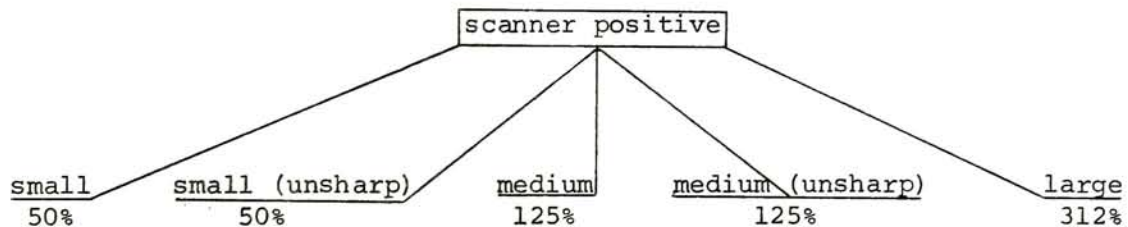


FIGURE 3D

For the changes in sharpness the position of the lens and copyboard are shifted slightly so that the magnification remains the same as the sharp reproduction but the focus is degraded.

The amount of degradation was determined visually by comparing isolated sections of both the sharp and unsharp halftones. In one case the sharp edge of a medium-sized halftone was compared with a sharp

edge on the small-sized halftone and the focus on the latter was degraded until the two appeared equal in sharpness. In the second case the alphanumeric target of a medium-sized halftone was defocused until it showed the same detail as a small-sized halftone.

Each of the fifty completed halftones could be characterized by their size and sharpness and each was checked for proper highlight and shadow dot area in a uniform area of the picture or gray scale using a Welch SS-100 dot area meter.

The final step in the procedure is the production of the photographic halftone prints. The gray scales and resolution targets were removed with a mask since they will not be useful in the subjective evaluation. The prints were then tray processed, dried, mounted, and trimmed leaving a 1/2-inch white border around each of the reproductions. The finished size of the image area in each print is listed in Figure 3E.

<u>Size</u>	<u>Width</u>	<u>Height</u>
Small	7.2cm	5.0cm
Medium	17.7cm	12.7cm
Large	42.4cm	30.6cm

FIGURE 3E

Figure 3F on the next page shows one set of small size reproductions.

1



2



3



4



5



6



7



8



9



10

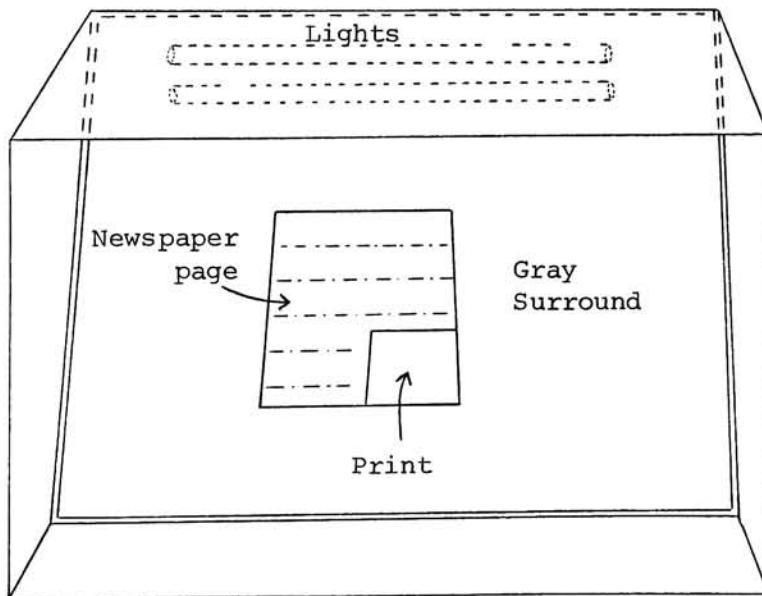
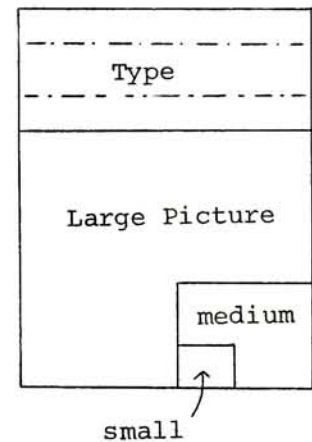


F. Subjective Evaluation

The next procedure was an evaluation process using observers to subjectively rate the reproductions for quality.

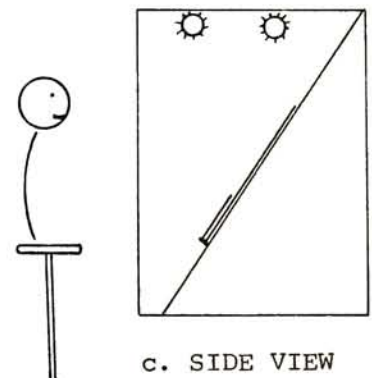
A large semi-enclosed Macbeth viewing booth with 5500°K fluorescent lamps is used as the viewing surround and method of illumination. A facsimile newspaper page produced on white photographic paper is used as the immediate background. The background is meant to be typical of the surround an observer would often see when viewing a printed picture. In this way any undesirable effects due to surround are minimized. Behind this newspaper page is a large gray cardboard held within the viewing booth at an angle to prevent glare at the observer position. Using this arrangement the reproductions are illuminated with 80-100 footcandles of light. The basic format is shown in the following diagrams.

a. PRINT LAYOUT



b. MACBETH VIEWING BOOTH

FIGURE 3G



c. SIDE VIEW

The picture quality may now be determined with a set of observers. It is possible to judge quality using many different methods. Ranking and paired comparison are among the most commonly used. In this project where angular subtense of the object is a primary concern, viewing more than one reproduction at a time may influence the results. This consideration limits subjective evaluation to one print at a time. When this is done, however, subtle differences between prints will not be as noticeable as if they were side by side. To reduce problems in this respect some sort of standard reference of fixed quality may be used for indirect comparison. The procedure would be to observe the standard and then the print for evaluation and then the standard and then the next print for evaluation and so on for the entire set. This would give a reference in the memory which would be reinforced periodically rather than lost and replaced with a new reference as might happen when a number of different prints are evaluated one after another.¹⁷ Each set in this project will be judged using one print from the set as a control reference.

The evaluation process was carried out with ten observers.

The following instructions were read to each observer:

"The object of the following procedure will be to determine the quality of each print in a series. A control print will be shown alternately with the print to be judged and each print will be rated against the control using the following scale:

0	↑	
10		definitely worse
20		noticeably worse
30		slightly worse
40		very nearly the same (toward worse)
50		<u>control picture</u>
60		very nearly the same (toward better)
70		slightly better
80		noticeably better
90		definitely better
100	↓	

The control print is not necessarily in the middle of the set and the rest of the prints will probably not be equally arranged above or below. Each print does not need a different rating, i.e. two prints may be rated 70 and so on.

Prints should be judged in a manner which will determine which print the observer personally prefers, ignoring such imperfections as dirt or fingerprints."

The prints were shown in a different order for each reproduction set and the same order was used for each observer. The control was the #1 print from the set being judged.

Ten Graphic Arts Research Center personnel were chosen to make the evaluations. They were not generally people accustomed to judging print quality in this manner. The group consisted of secretaries, pressmen, and several others.

The evaluations were fairly long and somewhat tedious, so to minimize the problem the small unsharp print was eliminated. Even with this concession the observations took between one-half and three-quarters of an hour.

The observers had similar feelings about the ratings. The lack of real criteria for the determination of quality was a common complaint. Apparently the criteria of "personal preference" is not adequate because as a group we have not been exposed to reproductions rated as either good or bad in order to form a frame of reference. The most common wording of this complaint is of the order, "I know the print is very different than the control, but I can't tell whether it's better or worse."

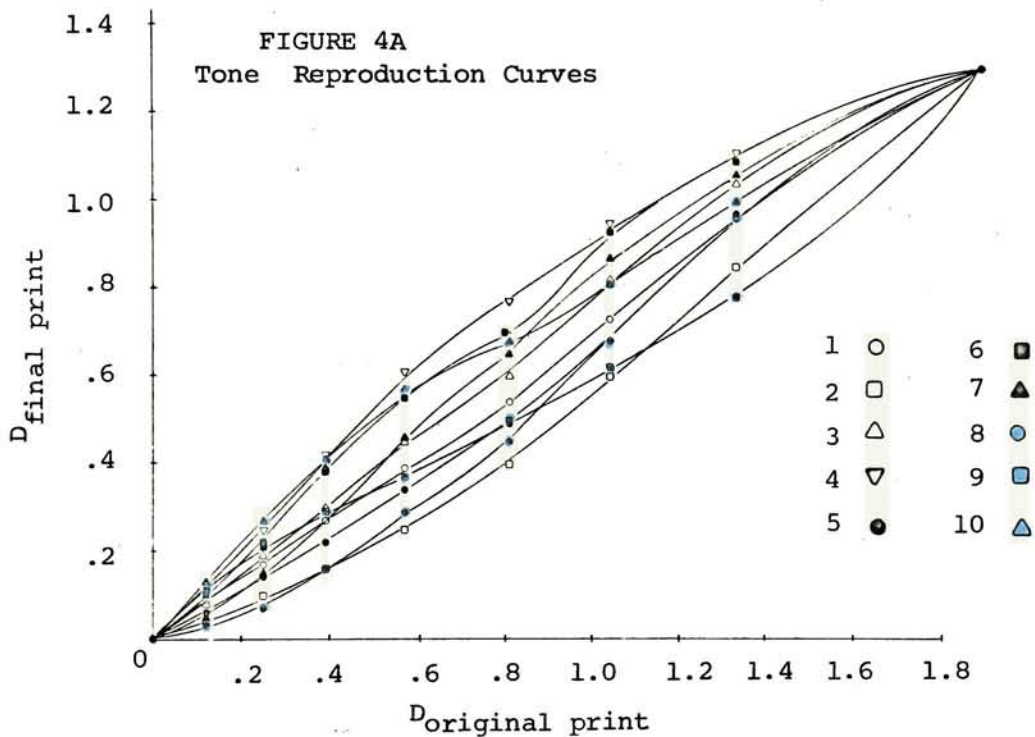
The ratings given by each observer are listed in Appendix C.

Section Four

Analysis and Results

From the data obtained from the prints and the observer quality ratings, an investigation of the relationship between preferred tone reproduction and picture size may be undertaken.

The tone reproduction curves for each print are shown below in Figure 4A.

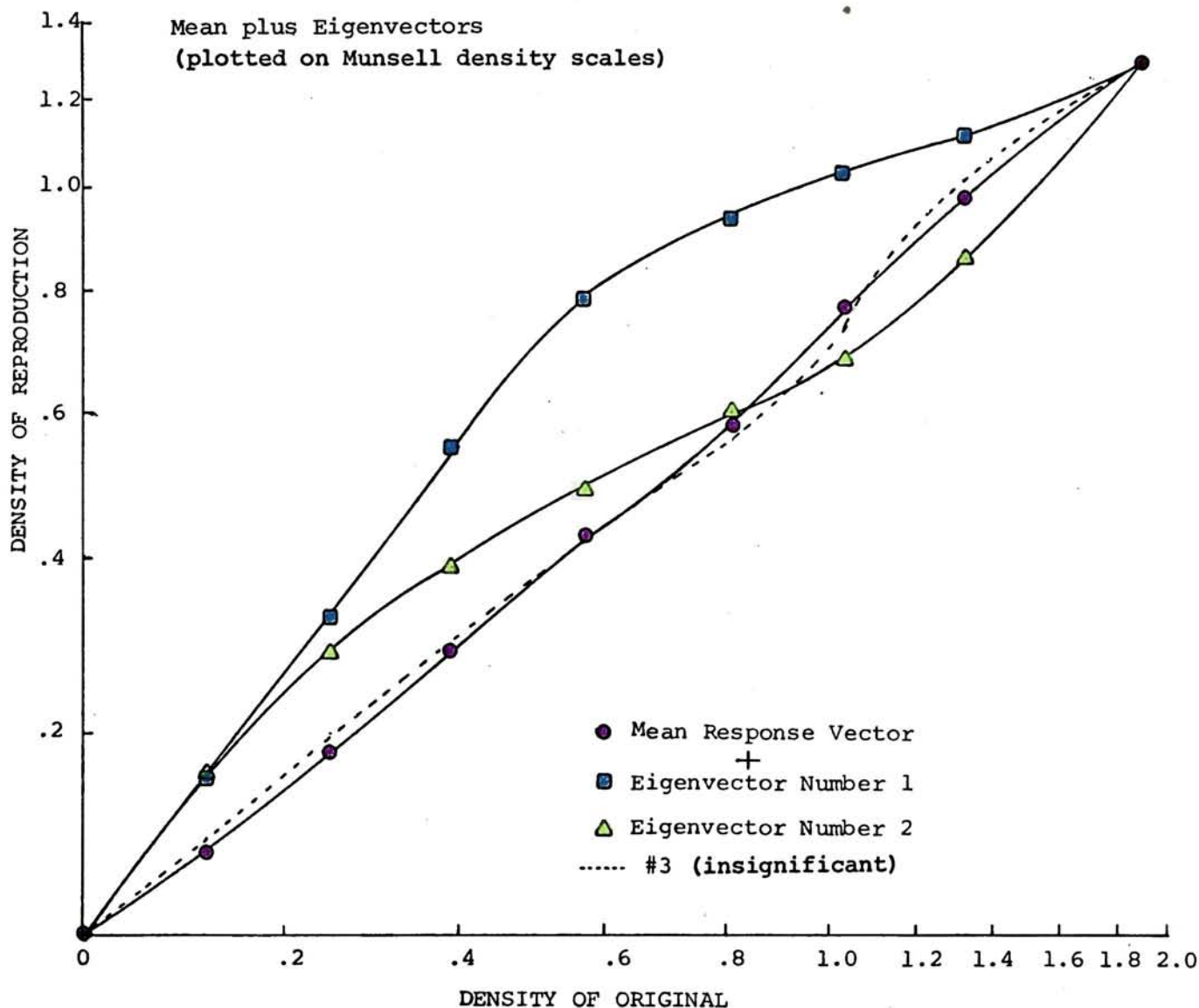


The range of curve shapes is quite substantial and this is expected from the visual difference between prints as seen in the previous section.

To determine the types of tone variations in the set, an eigenvector analysis (see Appendix B) is performed. There are two types of variation present; (a) the first, which accounts for 88% of the variation within the curves, is a bow-shaped curve. As the amount

of this vector increases in the reproduction, the print will become darker overall, and (b) the second, which accounts for an additional 11% of variation (99% total for both vectors), is an S-shaped curve. As it increases the reproduction becomes flatter and as it decreases the reproduction increases in contrast. These vectors are shown in Figure 4B.

FIGURE 4B



Any additional vectors do not account for a significant amount of variance in the original set.

Next, the observer quality ratings should be normalized so as to eliminate differences in observer rating standards and to allow all the observers to be considered as a single group.

The ratings were normalized so that for each observer the highest ranked print in a set equalled 100 and the lowest ranked print was 0. This was done with the formula:

$$R = (r - L) \frac{100}{H - L}$$

R = normalized rating
r = observer quality rating
H = highest rating
L = lowest rating

These normalized ratings were then averaged and the mean and standard deviation at each size was tabulated. These data are shown in Appendix C. Below are the mean values and their associated scalar multiples for each size.

Print	Scalar Multiples		Mean Normalized Ratings				
	λ_1	λ_2	\bar{R}_s	\bar{R}_m	\bar{R}_{mu}	\bar{R}_l	\bar{R}^*
1	-.096	.028	55.0	51.6	54.6	51.1	53.1
2	-.493	-.024	19.8	60.7	51.3	43.1	43.7
3	.097	-.148	56.2	80.1	63.9	65.8	66.5
4	.509	.013	46.9	35.6	37.2	35.4	38.8
5	-.225	-.140	76.8	67.2	70.8	77.8	73.2
6	.372	-.113	62.5	71.8	62.5	67.3	66.0
7	.160	-.354	70.5	74.9	77.2	51.5	68.5
8	-.336	-.388	45.8	71.9	46.5	57.4	55.9
9	-.270	.692	0.0	10.3	4.5	4.0	4.7
10	.282	.434	72.7	61.2	55.9	41.3	57.8

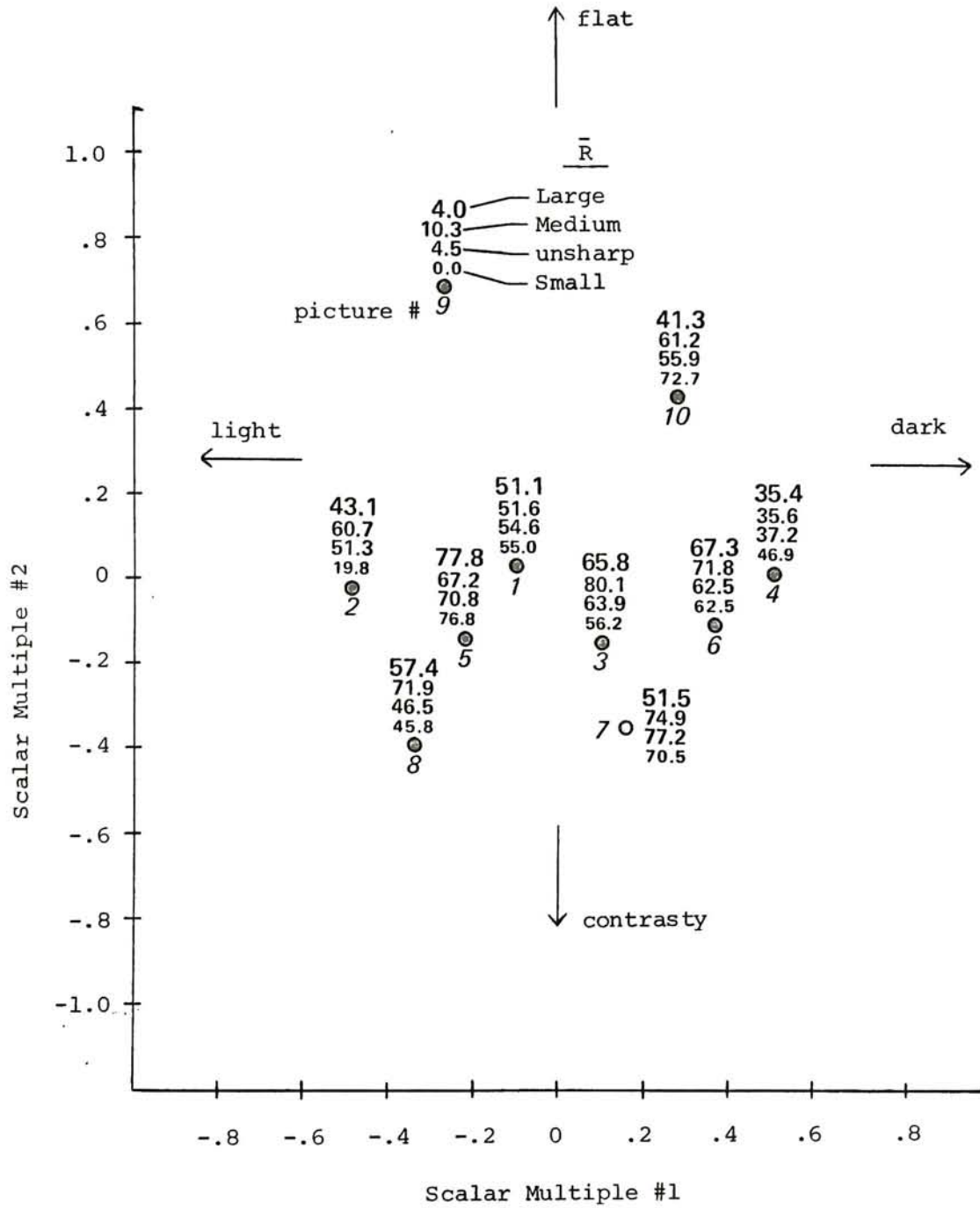
*At all points in this paper the common subscripts are:
s = small reproduction size
m = medium reproduction size
mu = medium (unsharp) reproduction size
l = large reproduction size

FIGURE 4C - Mean Normalized Quality Ratings

In addition the scalar multiples are plotted against the values for \bar{R} in Figure 4D.

FIGURE 4D

λ_1 and λ_2 vs. \bar{R} (mean normalized ratings)



At this stage it is reasonable to apply statistics to the data to determine the major sources of quality variance. Using an analysis of variance (ANOVA) the data variance due to the tested factors is compared with the data variance due to experimental error¹⁵ (see Appendix D). This procedure showed that, while tone reproduction did significantly affect quality ratings, neither picture size nor sharpness produced significant changes in the quality response when compared with those produced by error.

At this point at least two inferences may be drawn: (a) the data are as accurate as possible and the factors sharpness and reproduction size are actually insignificant in determining quality, or (b) the data were not sufficiently accurate and the role of the tested factors remains unknown.

Looking at the set of data from the observers in Appendix C, one sees large discrepancies within any single reproduction suggesting a system with a wide standard deviation. In such a case more quality decisions must be made to increase the accuracy of the calculation of a mean quality rating. Thus more observers or more observations would probably reduce experimental error. This in turn would increase the possibility of detecting significance of the tested factors.

Another method would be to smooth the quality ratings by fitting them to an appropriate curve. This would produce new and possibly more accurate data with which the experiment could be completed. The experiment will follow the latter course.

The method of least squares was used to relate the eigenvector scalar multiples to the normalized quality ratings. A quadratic curve of the form

$$Q = a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_1^2 + a_4\lambda_2^2 + a_5\lambda_1\lambda_2 + a_6$$

was chosen as the most reasonable model.

A computer program (see Appendix E) was created which would calculate the proper coefficients for each reproduction size and sharpness. The new mean normalized ratings calculated (\bar{R}_C) are shown in Figure 4E with the original \bar{R} for comparison.

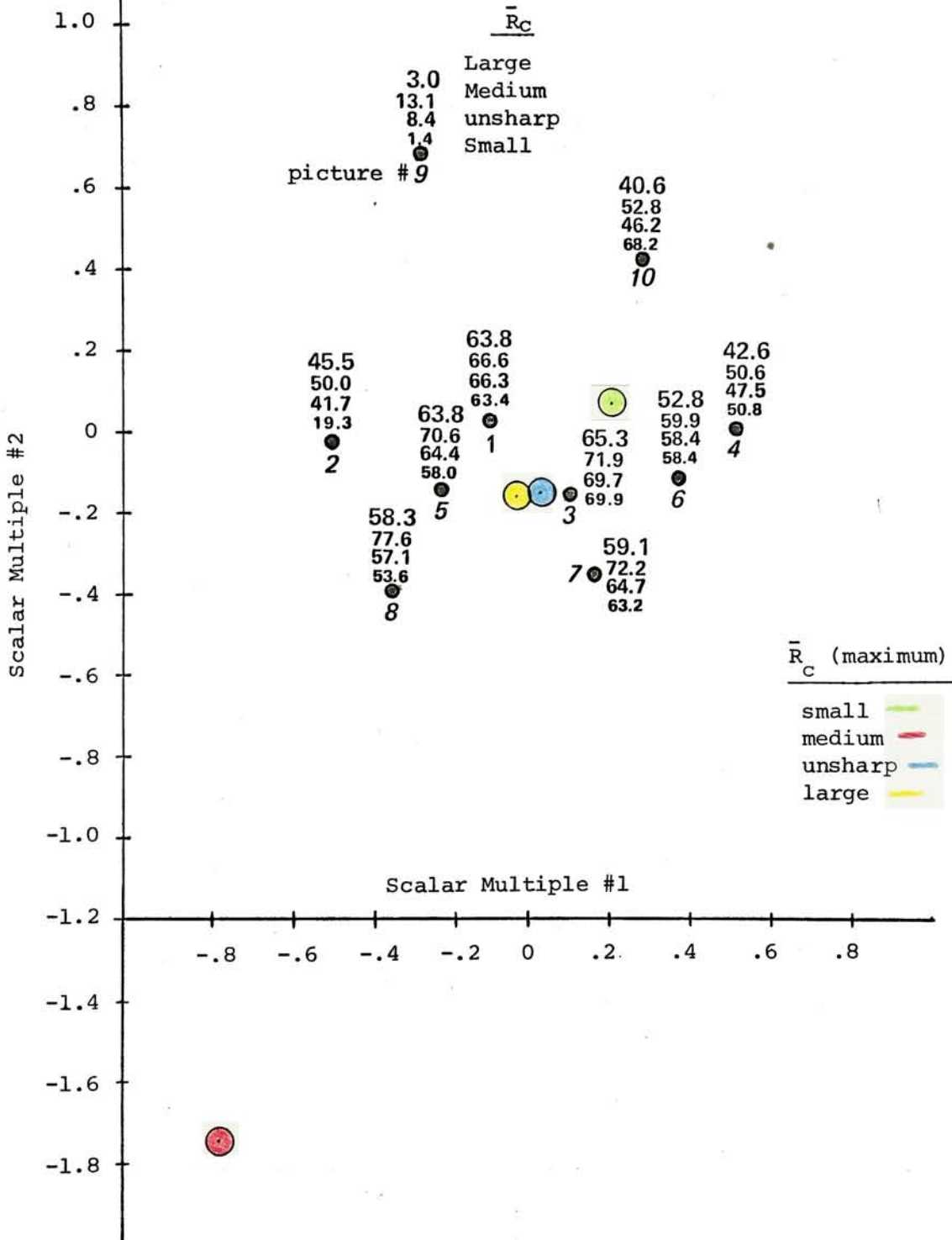
	<u>Small</u>		<u>Medium</u>		<u>Unsharp</u>		<u>Large</u>	
	\bar{R}	\bar{R}_C	\bar{R}	\bar{R}_C	\bar{R}	\bar{R}_C	\bar{R}	\bar{R}_C
1	55.0	63.4	51.6	66.6	54.6	66.3	51.1	63.8
2	19.8	19.3	60.7	50.0	51.3	41.7	43.1	45.5
3	56.2	69.9	80.1	71.9	63.9	69.7	65.8	65.3
4	46.9	50.8	35.6	50.6	37.2	47.5	35.4	42.6
5	76.8	58.0	67.2	70.6	70.8	64.4	77.8	63.8
6	62.5	58.4	71.8	59.9	62.5	58.4	67.3	52.8
7	70.5	63.2	74.9	72.2	77.2	64.7	51.5	59.1
8	45.8	53.6	71.9	77.6	46.5	57.1	57.4	58.3
9	0.0	1.4	10.3	13.1	4.5	8.4	4.0	3.0
10	72.7	68.2	61.2	52.8	55.9	46.2	41.3	40.6

FIGURE 4E - \bar{R} and Calculated \bar{R}_C

From this new information the tone reproduction needed for maximum quality for each size may be calculated as shown in Appendix E. The newly calculated \bar{R}_C and the points of maximum quality are plotted against the scalar multiples in Figure 4F below.

FIGURE 4F

λ_1 and λ_2 vs. \bar{R}_C (calculated ratings)



One would probably expect to find small differences in optimum tone reproduction and a smooth progression from small to large. The location of the maximum quality point for the medium sized reproduction indicates immediately that something is amiss. Calculating the maximum values of \bar{R}_C we find that they are:

Size	Small	Medium	Unsharp	Large
\bar{R}_C maximum	70.9	97.1	66.0	70.3

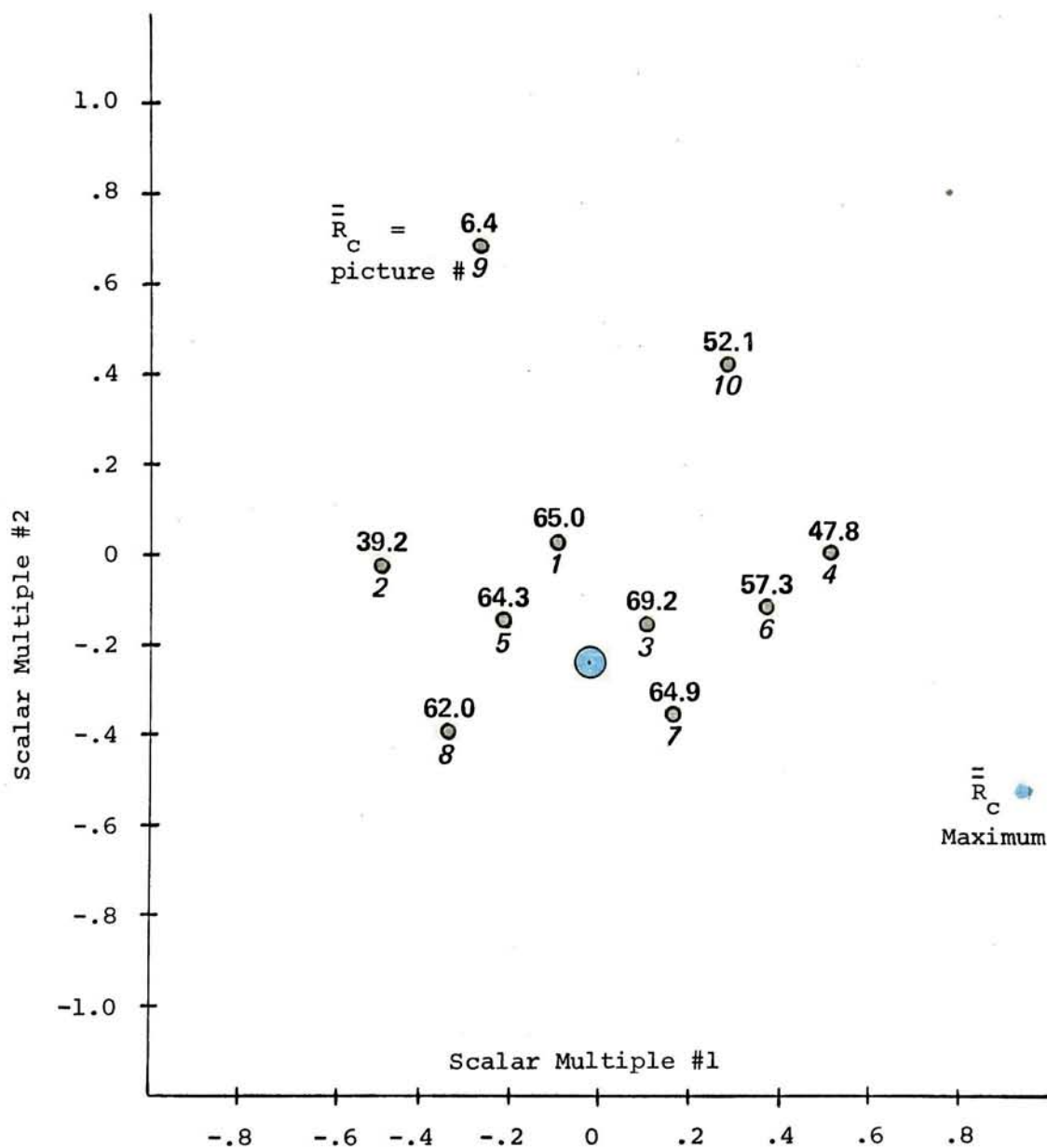
FIGURE 4G - Values for \bar{R}_C Maximum

There is no apparent reason why the medium reproduction should have a maximum \bar{R}_C so unlike the other reproductions. Looking again at Figure 4F, the values of \bar{R}_C continue to increase as you approach #8 from the tested areas. Thus the maximum would be expected to appear outside the range that was tested. In such a case the quadratic curve fitting was forced to extrapolate an answer and an incorrect result was drawn. This is a result of studying too limited a tone reproduction range and more observations would not result in a correct answer unless the ratings were changed considerably. At this point the experiment cannot be done again and more accurate results will await future experimentation in this area.

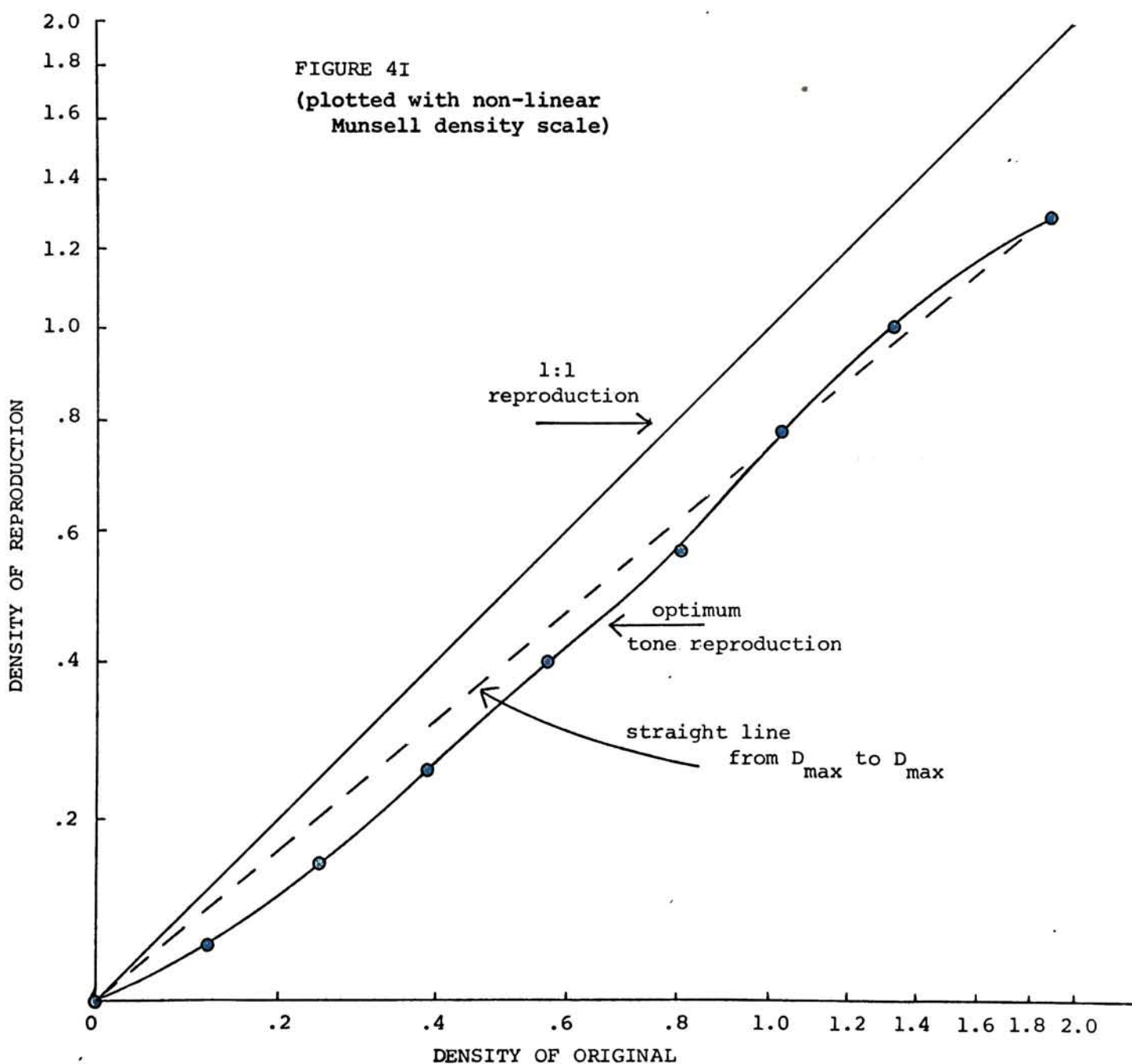
If all of the data from the original normalized observations are combined, information about the optimum tone reproduction for this subject (not considering size a factor) may be obtained. This information should be quite accurate because of the increased amount of data used to determine it. A plot of λ_1 , λ_2 , and \bar{R}_C is shown in Figure 4H along with the position of maximum quality.

FIGURE 4H

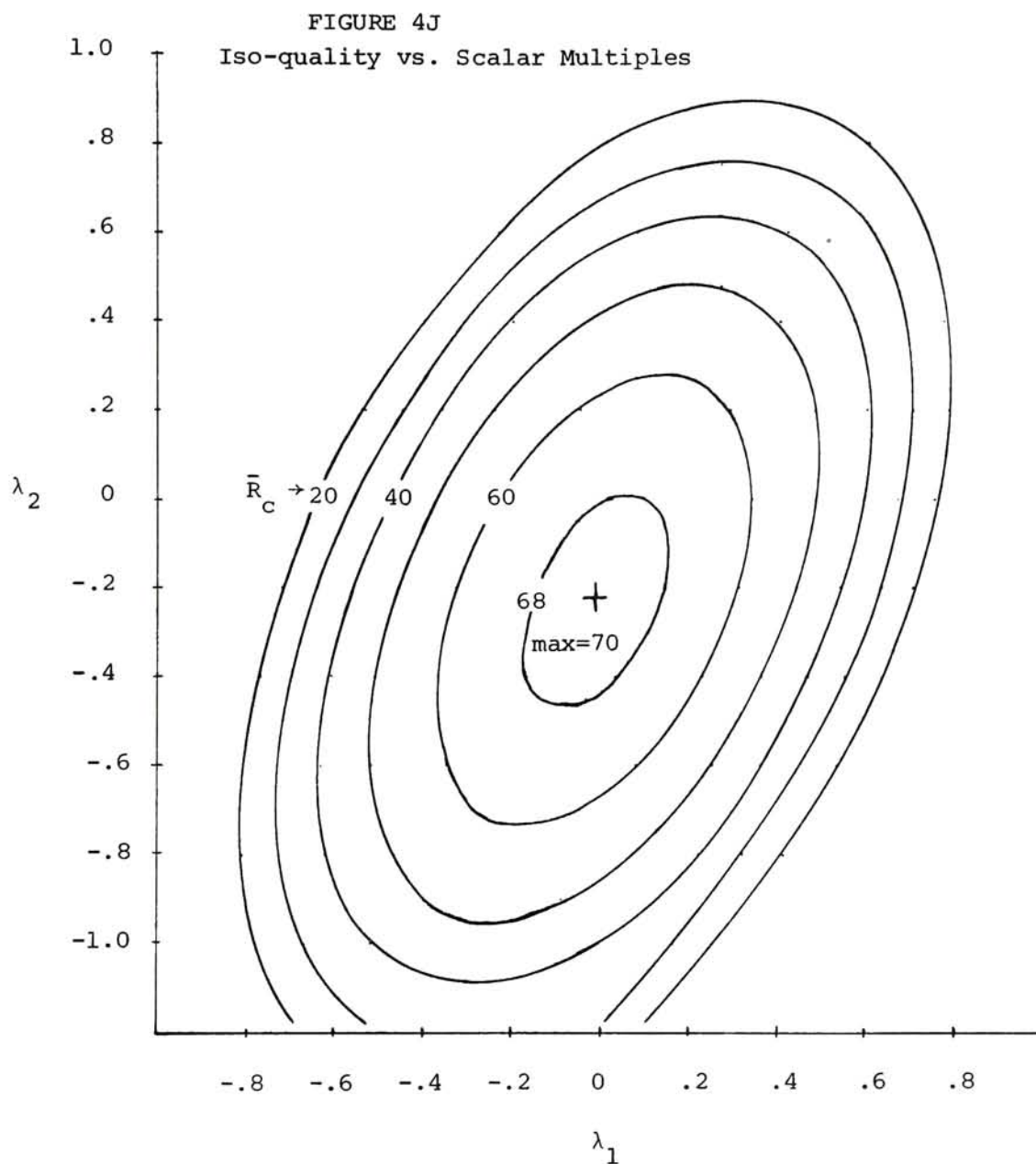
λ_1 and λ_2 vs. \bar{R}_C (average calculated ratings)



This optimum tone reproduction is then plotted using RIT TR graph paper with axes scaled according to visual density perception. Figure 4I shows that this optimum reproduction has more contrast than a straight line reproduction would have. Overall such an optimum print is lighter than the corresponding straight line reproduction, also.



In addition using the quadratic model, ellipses showing regions of equal quality may be drawn. In Figure 4J this plot of iso-quality vs. scalar multiples is shown.



The ellipses for any size print would be similar to these and such a diagram is extremely useful in choosing a tone reproduction range to be studied.

At this point the procedures are established, computer programs are available, and the tone reproduction range has been narrowed to a useful size. Using this as a starting point I hope that another project will be initiated to gain more concrete results and perhaps investigate similar questions on optimum tone reproduction.

An interesting project, perhaps related to similar work in this subject, would be to determine the correlation between the three gradation knob settings of the scanner and the resultant curve shapes. Since only two types of variation (s-shaped and bowed) seem to be available, only two control knobs are really needed.

Section Five

Conclusions

Due to deficiencies in the tone reproduction range studied and the wide quality response range given by the observers, no conclusive information on the preferred tone reproduction for prints of different sizes may be made.

Several observations may be drawn from the work done in the project, however.

First is the question of quality preference in a group of observers. Is there really a tone reproduction which most observers will consider to produce a high quality print? It is much easier for a person to determine sharpness, contrast, lightness or darkness and other single factors than it is to bring them all together in a subjective quality judgement. In this experiment the observers expressed widely different quality preferences and showed little ability to make firm quality decisions. If quality is not a more definite criterion that it has shown itself to be, then perhaps small changes in quality due to size or sharpness are not really important after all. An interesting project would be to find what factors (contrast, sharpness, darkness, etc.) actually contribute most to quality.

Secondly, the application of eigenvectors to characterize tone reproduction curves has proven to be an extremely useful method. This could be used to aid in the analysis of any study using variations of tone reproduction. It is particularly adaptable to mathematical analysis of any multivariate system.

Third, the results obtained from the study of the averaged data, from all observations on all prints, has given much insight into the relationship of tone reproduction change to quality preference. The optimum tone reproduction curve for an average of different sized prints should be of value in a practical reproduction system.

All that seems to remain at this point to obtain meaningful information is the selection of a wider or, at least, more appropriate range of tone reproductions and the use of a larger observer group and/or employment of a more efficient judging system.

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APPENDICES

APPENDIX A

Processing and Production

APPENDIX A

Processing and Production

I. Scanner Procedures

Scanner - RCA (Hell) Chromograph C286

Development - D-8 developer 2:1

3 minutes @ 68°F RIT agitation

Film - Dupont Scanner Film

Several adjustments were performed on the scanner. Prior to these, however, development tests were carried out. The developer chosen was D-8 for its ability to give high contrast and low fog with the Dupont continuous tone scanner film. At the same time the iris diaphragm in the exposing section of the scanner was adjusted to give a sufficient exposure to the scanner film without resorting to excessive voltage at the exposing lamp.

These adjustments resulted in an iris opening of 4 1/4 units and development in D-8 (2 pts. developer: 1 pt. water) at 68°F for 3 minutes.

Next the linearization of the scanner output was performed. This procedure should produce a linear relationship between voltage applied to the exposing lamp and the density produced on the scanner positive. A straight line relationship was not obtained but the results were adequate for the range of densities to be produced.

Next a sampling of the range of gradation was made and one further need was determined. The black adjustment control, which

balances the output of the photomultipliers on a black shadow area, will significantly decrease the effectiveness of the gradation controls if it is set on too low a density. The setting found to be best was on a density .15 above the actual shadow density.

Other than this all single color adjustment controls were set normally and all color correction and special tone controls were turned off.

II. Halftone Films

Camera - Klimsch copy camera

Film - Kodak Ortho Type 3 #2556

Processing - Log E automatic processor

Kodak RT developer and replenisher

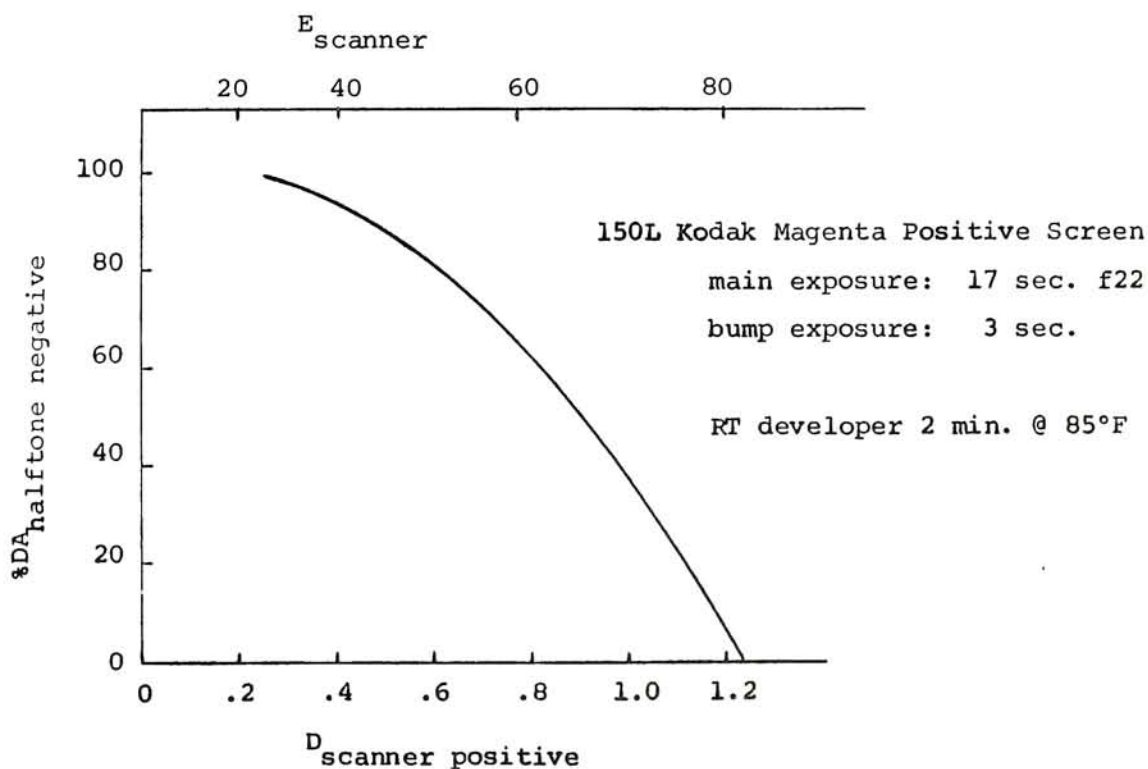
2 minutes @ 85°F

Screen - Kodak Magenta Positive 150%/in.

With the scanner positive in the transparency holder of the Klimsch (illuminated from behind with pulsed xenon lamps), the image size and sharpness could be changed to produce the needed halftones. The copyboard was carefully masked to prevent as much flare light from reaching the lens as possible.

Main and bump exposures were used in different ratios for small BDR corrections. Exposures were: main = $16 \frac{1}{2}$ sec \pm 1 $\frac{1}{2}$ sec

bump = 2.7 sec \pm .5 sec
(.7ND filter over lens)



III. Halftone Prints

Paper - Kodabromide E-2

Exposure - Kodak point source, tap 2, 24 sec with 1.2ND

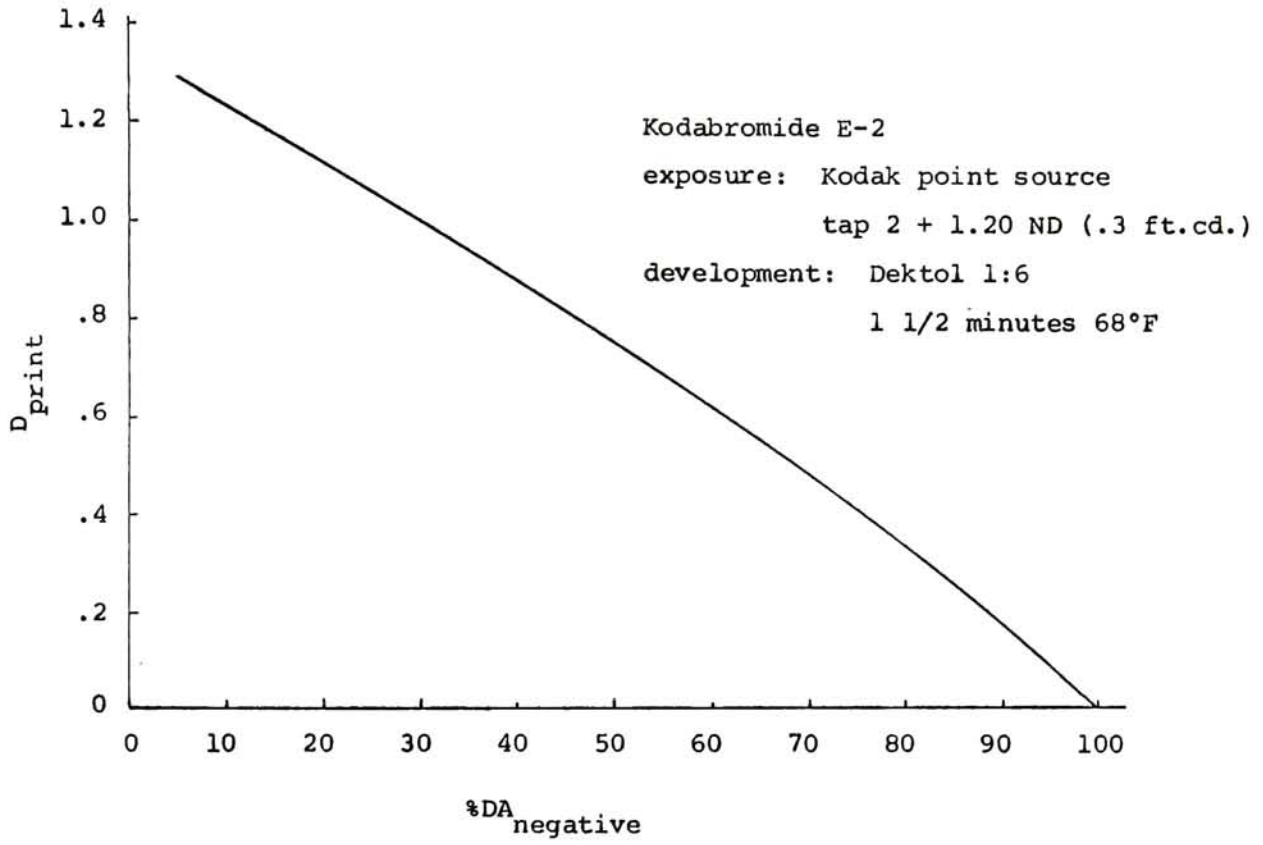
Development - D-72 1:6 for 1 1/2 minutes @ 68°F

The photographic halftone prints were produced by exposing the Kodabromide paper in vacuum contact with the masked film halftone. The Klimsch film back was used as an open-faced vacuum board and a point light source at a distance of 52" from the back was used for exposure.

The exposure remained constant for each print and development varied only in the amount of developer used for different size prints. Fresh developer was used for each print except for the small size where two were processed at a time. The dilution of the developer caused some difficulty in control of maximum density levels and some prints were made over to correct for errors. Maximum density was maintained to $\pm .03$.

The prints were stopped and fixed normally and hypo clearing agent was used to reduce washing time. The prints were dried in a rotary drum drier and mounted on cardboard with dry mounting tissue.

A graph of % dot area vs. print density is shown on the next page.



APPENDIX B

Eigenvector Analysis

APPENDIX B

Eigenvector Analysis

I. Mathematical Basis

If n sets of response data Z are available at m levels of a variable X , then the response vector may be arranged in a data matrix of n rows and m columns.

From this data matrix a covariance matrix (W) and a mean vector (μ) may be calculated from the original set of data.

The mean vector (μ) is the mean value of each of the n sets of data at a given level of X . It supplies a method of specifying the location of the family of curves to be analysed.

The covariance matrix (W) is defined as $W = f(z - \mu)(z - \mu)'$.^{*} This matrix contains a diagonal element, $f(z_i - \mu_i)^2$, which is the variance of Z , and a nondiagonal element, $f(z_i - \mu_i)(z_j - \mu_j)$, which is the covariance of z_i and z_j . These terms are often referred to as sums of squares and sums of cross products and may be used as a measure of correlation between data groups.

From this covariance matrix the vector and root corresponding to maximum variance may be calculated. The equation for the root is $|W - \lambda I|$ ^{**} = 0 and for the vector $WV = \lambda V$ or $(W - \lambda I)V = 0$.

In the calculation of the latent root for a matrix of order n , n roots will be calculated. The largest of these corresponds to the largest variance.

^{*} The notation A' indicates the transpose of a matrix A (the transpose of a matrix has the rows and columns interchanged, i.e. $A_{ij} = A'_{ji}$).

^{**} I is the identity matrix which contains a diagonal row of 1's. This matrix is equal to 1 in any matrix algebra manipulations.

In an example where $n = 2$, the following equation is obtained:

$$\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix} - \lambda_i \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 0 = \begin{vmatrix} z_{11}^{-\lambda_i} & z_{12}^{-0} \\ z_{21}^{-0} & z_{22}^{-\lambda_i} \end{vmatrix}$$

$$(z_{11}^{-\lambda_i}) (z_{22}^{-\lambda_i}) - (z_{12}) (z_{21}) = 0$$

and

$$\lambda_i^2 - z_{11}\lambda_i - z_{22}\lambda_i + z_{11}z_{22} - z_{21}z_{12} = 0 \quad .$$

This equation may then be solved for its roots.

In a computer application this solution is most easily obtained in an iterative procedure in which the covariance matrix is premultiplied by normalized approximations of the final vector until the iteration converges on the final solution of the roots.

The largest root is then used in the equation $(W - \lambda I)V = 0$ to find the component vector, V , associated with that root.

The number of vectors and roots necessary to account for a given percentage of original variability may be determined. The sum of the diagonal elements of W is called the ^ttrace and is equal to the sum of all the roots of W . The ratio of the sum of the roots calculated to the trace is a measure of the variability explained.

The scalar multiples may then be normalized with a weighting factor (f) obtained by dividing the elements of V by the root and

$$\text{summing the product of } \lambda = \sum_{i=1}^n f(z_i - \bar{z}_i).$$

Thus we can obtain a complete specification of the original data set in fewer parameters. The mean (μ), the component vector (V), and the scalar multiple (λ) may be used to specify the curves in the original family.

Any response could then be stated mathematically as follows:

$$z_1 = \bar{z}_1 + \lambda_1 v_{1,1} + \lambda_2 v_{2,1} + \lambda_3 v_{3,1} + \dots$$

$$z_2 = \bar{z}_2 + \lambda_1 v_{1,2} + \lambda_2 v_{2,2} + \lambda_3 v_{3,2} + \dots$$

$$z_r = \bar{z}_r + \lambda_1 v_{1,r} + \lambda_2 v_{2,r} + \lambda_3 v_{3,r} + \dots$$

NOTE: It should be understood that eigenvectors are sets of discrete points. For illustration they may be represented by a curve drawn through those points.

APPENDIX B

B. Computer Program to Compute Mean, Eigenvectors, and Scalar Multiples from a Data Set

IASSIGN M:SI,(FILE,EIGN)

IFORTTRAN

OPTIONS:

EXTENDED FIV-H VERSION DOO

```

1: C      I=ROWS(VARYING ORDINATE)  J=COLUMNS(CONSTANT STEP
2: C      TO CHANGE SIZE OF DATA ARRAY THE FOLLOWING STEPS
3: C      MUST BE CHANGED -- #8,9,10 --12,13-- #80,81
4: C      DIMENSION A(I,J),P(I,J),W(J,J),PTP(J,J)
5: C      DIMENSION XMEAN(1,J),U(1,J),Y1(1,J),Y(1,J),V(1,J)
6: C      DIMENSION WW(J,1,J),X(I*J),XX(J*J)
7: C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
8: C      DIMENSION A(10,9),P(10,9),W(9,9),PTP(9,9)
9: C      DIMENSION XMEAN(1,9),U(1,9),Y1(1,9),Y(1,9)
10: C     DIMENSION V(1,9),WW(10,1,9),X(10,9),XX(9,9)
11: C     PRINT 30
12: C     I=10
13: C     J=9
14: C     QQ=0.0
15: C     READ 19,A
16: C     DO 3 K=1,J
17: C     SUM=0.0
18: C     DO 2 L=1,I
19: C     SUM=A(L,K)+SUM
20: C     2 CONTINUE
21: C     MEAN ROW VALUE
22: C     XMEAN(1,K)=SUM/I
23: C     3 CONTINUE
24: C     PRINT 27
25: C     PRINT 17,XMEAN
26: C     PRINT 26
27: C     DO 4 K=1,J
28: C     DO 4 L=1,I
29: C     MEAN CORRECTED DATA MATRIX
30: C     P(L,K)=A(L,K)-XMEAN(1,K)
31: C     4 CONTINUE
32: C     TRACE=0.0
33: C     PREMULIPLY 'P' BY ITS TRANSPOSE
34: C     100 DO 15 L=1,I
35: C     DO 15 K=1,J
36: C     X(K,L)=P(L,K)
37: C     15 CONTINUE
38: C     DO 16 K=1,J
39: C     DO 16 L=1,J
40: C     PTP(K,L)=0.0
41: C     DO 16 M=1,I
42: C     PTP(K,L)=PTP(K,L)+X(K,M)*P(M,L)

```



```

3:      10 CONTINUE
44:      DO 25 L=1,J
45:      DO 25 LL=1,J
46:      W(LL,L)=PTP(LL,L)
47:      90 CONTINUE
48:      DO 200 N=1,10
49:      IF (N .LE. 2) GO TO 50
50:      DO 49 L=1,J
51:      DO 49 LL=1,J
52:      W(LL,L)=PTP(LL,L)
53:      49 CONTINUE
54:      50 IF (N .LT. 2) GO TO 94
55:  C      PREMULIPLY 'V' BY ITS TRANSPOSE
56:      DO 32 L=1,J
57:      XX(L,1)=V(1,L)
58:      32 CONTINUE
59:      DO 31 L=1,J
60:      DO 31 K=1,J
61:      PTP(K,L)=XX(K,1)*V(1,L)
62:      31 CONTINUE
63:      DO 94 L=1,J
64:      DO 94 LL=1,J
65:      PTP(LL,L)=W(LL,L)-PTP(LL,L)
66:      94 CONTINUE
67:      DO 5 L=1,J
68:      IF (N .GE. 2) GO TO 98
69:  C      TRACE IS SUM OF DIAGONAL ROW OF PTP
70:      TRACE=PTP(L,L)+TRACE
71:      98 U(1,L)=1.0
72:      Y1(1,L)=1.0
73:      5 CONTINUE
74:      DO 8 K=1,100
75:      DO 35 L=1,J
76:      Y(1,L)=0.0
77:      DO 35 M=1,J
78:      Y(1,L)=U(1,M)*PTP(M,L)+Y(1,L)
79:      35 CONTINUE
80:      Q=DMAX1(Y(1,1),Y(1,2),Y(1,3),Y(1,4),Y(1,5),
81:      1Y(1,6),Y(1,7),Y(1,8),Y(1,9))
82:      DO 6 L=1,J
83:      R=ABS(Y1(1,L)-Y(1,L))
84:      IF (R .GE. .000000001) GO TO 7
85:      6 CONTINUE
86:      GO TO 8
87:      7 CONTINUE
88:      DO 70 M=1,J
89:      Y1(1,M)=Y(1,M)
90:      70 CONTINUE
91:      DO 8 L=1,J
92:      U(1,L)=Y(1,L)/Q
93:      8 CONTINUE
94:      QQ=Q+QQ
95:      SUM=0.0
96:      DO 10 L=1,J
97:      SUM=Y(1,L)**2.0+SUM
98:      10 CONTINUE
99:      Z=DSQRT(Q/SUM)
100:      DO 11 L=1,J
101:      V(1,L)=Y(1,L)*Z

```

```

1
02:      WW(N,1,L)=(Y(1,L)*Z)/Q
103:    11 CONTINUE
104:      EXP=QQ/TRACE
105:      PRINT 28,N
106:      PRINT 23,V
107:      PRINT 24,EXP
108:      IF (EXP .GT. .9999) GO TO 75
109:      PRINT 29
110:      DO 79 L=1,I
111:      COEF=0.0
112:      DO 69 K=1,J
113:      COEF=WW(N,1,K)*P(L,K)+COEF
114:    69 CONTINUE
115:      PRINT 25,L,COEF
116:    79 CONTINUE
117:      PRINT 26
118:    200 CONTINUE
119:      17 FORMAT (5F10.4)
120:      19 FORMAT (F8.7)
121:      23 FORMAT (5F10.5)
122:      24 FORMAT (16HTRACE EXPLAINED=,F6.5,/)
123:      25 FORMAT (1H(,12,1H),2X,F7.4)
124:      26 FORMAT(///)
125:      27 FORMAT(10X,20HMEAN RESPONSE VECTOR )
126:      28 FORMAT (10X,23HCHARACTERISTIC VECTOR #,11)
127:      29 FORMAT (16HSCALAR MULTIPLES)
128:      30 FORMAT (5X,37HEIGENVECTOR ANALYSIS OF RESPONSE DATA,///)
129:      75 END

```

SUBPROGRAMS

4MAIN	F:105	F:106	F:108	9INITIAL	M:DO
M:OC	9PRINT	9ENDIOL	9RTOD	9READ	9IOLUSA
9ITOD	DMAX1	ABS	9POWDR	DSQRT	9IODATA
9STOP					

PROGRAM ALLOCATION

2A0.0	I	2A1.0	J	2A2.0	QQ	2A4.0	K
2A6.0	SUM	2A8.0	L	2AA.0	TRACE	2AC.0	M
2AD.0	LL	2AE.0	N	2B0.0	Q	2B2.0	R
2B4.0	Z	2B6.0	EXP	2B8.0	COEF		
2BA.0	A	36E.0	P	422.0	W	4C4.0	PTP
566.0	XMEAN	578.0	U	58A.0	Y1	59C.0	Y
5AE.0	V	5C0.0	WW	674.0	X	728.0	XX

PROGRAM SIZE 7CA

PROGRAM END

!LOAD
 ELEMENT FILES:
 OPTIONS:
 F:105=DATA2
 F:

SEV.LEV. = 0
 XEQ? Y

EIGENVECTOR ANALYSIS OF RESPONSE DATA

MEAN RESPONSE VECTOR
 .0000 .0310 .1730 .2900 .4280
 .5730 .7690 .9820 1.3000

CHARACTERISTIC VECTOR #1
 .00000 .06905 .14693 .25982 .35843
 .36067 .36178 .26023 .00000
 TRACE EXPLAINED=.87936

SCALAR MULTIPLES

(1) -.0960
 (2) -.4928
 (3) .0974
 (4) .5037
 (5) -.2247
 (6) .3719
 (7) .1597
 (8) -.3362
 (9) -.2698
 (10) .2318

CHARACTERISTIC VECTOR #2
 .00000 .07481 .11142 .10492 .06450
 .01537 -.09464 -.16609 .00000
 TRACE EXPLAINED=.99100

SCALAR MULTIPLES

(1) .0235
 (2) -.0243
 (3) -.1435
 (4) .0126
 (5) -.1325
 (6) -.1127
 (7) -.3542
 (8) -.3383
 (9) .6925
 (10) .4340

CHARACTERISTIC VECTOR #3

.00000	.01482	.01887	.00506	.00907
-.02098	-.03312	.04299	.00000	

TRACE EXPLAINED=.99750

SCALAR MULTIPLES

(1)	.1671
(2)	-.1707
(3)	.1977
(4)	-.0676
(5)	.5154
(6)	-.1506
(7)	-.5408
(8)	.0424
(9)	-.3943
(10)	.4015

CHARACTERISTIC VECTOR #4

.00000	.01186	.01053	.00519	-.01134
-.01707	.01863	-.00091	.00000	

TRACE EXPLAINED=.99917

SCALAR MULTIPLES

(1)	.2735
(2)	.0426
(3)	.4125
(4)	-.1302
(5)	-.1533
(6)	.5457
(7)	-.4238
(8)	-.1923
(9)	.0741
(10)	-.4109

APPENDIX C
Quality Ratings

APPENDIX C

Observer Quality Ratings (r)

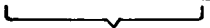
Observer	PRINT 2				PRINT 3				PRINT 4				PRINT 5				PRINT 6			
	S	M	MU	L	S	M	MU	L	S	M	MU	L	S	M	MU	L	S	M	MU	L
1	35	30	35	40	35	40	55	60	40	20	30	35	40	75	65	40	60	60	45	65
2	30	20	60	40	30	70	70	70	90	70	80	60	80	30	30	80	60	80	40	60
3	20	65	60	40	65	65	45	75	55	20	35	40	55	60	75	75	40	45	65	55
4	40	80	55	60	45	55	45	45	35	40	30	40	60	80	60	75	55	40	30	38
5	30	80	75	35	60	80	40	25	15	20	0	10	85	60	80	70	40	20	15	20
6	10	30	20	20	80	85	90	80	70	80	70	70	40	35	40	40	70	90	90	80
7	30	40	20	60	60	55	70	60	40	60	40	30	80	70	60	40	40	70	60	70
8	40	40	30	80	60	80	40	40	30	20	20	30	70	70	60	60	80	80	60	60
9	30	80	70	30	40	80	30	60	30	30	20	30	90	70	80	90	30	80	90	60
10	20	80	60	30	40	80	80	70	70	60	80	80	40	30	40	80	60	80	60	80

Observer	PRINT 7				PRINT 8				PRINT 9				PRINT 10			
	S	M	MU	L	S	M	MU	L	S	M	MU	L	S	M	MU	L
1	45	60	60	45	40	35	35	40	10	10	10	15	60	65	45	60
2	80	80	70	40	30	70	40	70	10	10	30	10	80	40	60	30
3	70	60	60	60	40	45	65	70	25	30	30	25	60	70	60	45
4	45	40	50	45	50	80	75	60	30	30	30	35	47	55	45	47
5	70	30	70	60	45	80	25	35	10	10	20	10	40	65	35	45
6	60	85	70	80	20	45	20	10	10	10	10	10	90	90	90	40
7	70	80	70	40	30	40	20	60	10	10	10	30	60	40	40	40
8	30	50	60	40	55	70	30	60	30	20	30	40	40	80	40	55
9	80	90	40	40	40	90	60	40	20	80	20	20	70	30	40	40
10	70	90	80	60	80	70	70	70	20	10	20	40	80	20	60	40

NOTE: PRINT #1 used as control from each set ($r = 50$).

Each reproduction size was evaluated in a different order, the orders remained constant for each observer.

order →		1	2	3	4	5	6	7	8	9
A	medium	5	10	3	2	4	9	8	6	7
B	small	6	4	10	3	9	7	8	2	5
C	large	10	2	9	6	7	8	4	3	5
D	medium (unsharp)	9	3	10	7	8	2	4	5	6



 print numbers

APPENDIX C

Normalized Quality Ratings (R)Small Print

Observer	Print 1	2	3	4	5	6	7	8	9	10
1	80.0	50.0	50.0	60.0	60.0	100.0	70.0	60.0	0.0	100.0
2	50.0	25.0	25.0	100.0	87.5	62.5	87.5	25.0	0.0	87.5
3	60.0	0.0	90.0	70.0	70.0	40.0	100.0	40.0	0.0	80.0
4	66.7	33.3	50.0	16.7	100.0	83.3	50.0	66.7	0.0	56.7
5	53.3	26.7	66.7	6.7	100.0	40.0	80.0	46.7	0.0	40.0
6	50.0	0.0	87.5	75.0	37.5	75.0	62.5	12.5	0.0	100.0
7	57.1	28.6	71.4	42.9	100.0	42.9	85.7	28.6	0.0	71.4
8	40.0	20.0	60.0	0.0	80.0	100.0	0.0	50.0	0.0	20.0
9	42.9	14.3	28.6	14.3	100.0	14.3	85.7	28.6	0.0	71.4
10	50.0	0.0	33.3	83.3	33.3	66.7	83.3	100.0	0.0	100.0
\bar{R}	55.0	19.79	56.22	46.89	76.83	62.47	70.47	45.81	0.0	72.70
s	11.74	16.51	23.66	35.66	25.91	28.16	28.54	25.30	0.0	27.11

Medium Print

Observer	Print 1	2	3	4	5	6	7	8	9	10
1	61.5	30.8	46.2	15.4	100.0	76.9	76.9	38.5	0.0	84.6
2	57.1	14.3	85.7	85.7	28.6	100.0	100.0	85.7	0.0	42.9
3	60.0	90.0	90.0	0.0	80.0	50.0	80.0	50.0	20.0	100.0
4	40.0	100.0	50.0	20.0	100.0	20.0	20.0	100.0	0.0	50.0
5	57.1	100.0	100.0	14.3	71.4	14.3	28.6	100.0	0.0	78.6
6	50.0	25.0	93.8	87.5	31.3	100.0	93.8	43.8	0.0	100.0
7	57.1	42.9	64.3	71.4	85.7	85.7	100.0	42.9	0.0	42.9
8	50.0	33.3	100.0	0.0	83.3	100.0	50.0	83.3	0.0	100.0
9	33.3	83.3	83.3	0.0	66.7	83.3	100.0	100.0	83.3	0.0
10	50.0	87.5	87.5	62.5	25.0	87.5	100.0	75.0	0.0	12.5
\bar{R}	51.61	60.71	80.08	35.65	67.20	71.77	74.93	71.92	10.33	61.15
s	9.05	34.26	19.66	36.71	28.86	32.40	31.06	25.66	26.40	36.94

Normalized Quality Ratings (R) - ContinuedMedium Unsharp Print

Observer	Print									
	1	2	3	4	5	6	7	8	9	10
1	72.7	45.5	81.8	36.4	100.0	63.6	90.9	45.5	0.0	63.6
2	40.0	60.0	80.0	100.0	0.0	20.0	80.0	20.0	0.0	60.0
3	44.4	66.7	33.3	11.1	100.0	77.8	66.7	77.8	0.0	66.7
4	44.4	55.6	33.3	0.0	66.7	0.0	44.4	100.0	0.0	33.3
5	60.0	93.3	46.7	0.0	100.0	13.3	86.7	26.7	20.0	40.0
6	50.0	12.5	100.0	75.0	37.5	100.0	75.0	12.5	0.0	100.0
7	66.7	16.7	100.0	50.0	83.3	83.3	100.0	16.7	0.0	50.0
8	75.0	25.0	50.0	0.0	100.0	100.0	100.0	25.0	25.0	50.0
9	42.9	71.4	14.3	0.0	85.7	100.0	28.6	57.1	0.0	28.6
10	50.0	66.7	100.0	100.0	33.3	66.7	100.0	83.3	0.0	66.7
<hr/>										
\bar{R}	54.61	51.34	63.94	37.25	70.83	62.47	77.23	46.46	4.50	55.89
s	12.99	26.17	32.16	41.76	35.58	38.06	24.49	31.46	9.56	20.64

Large Print

Observer	Print									
	1	2	3	4	5	6	7	8	9	10
1	70.0	50.0	90.0	40.0	50.0	100.0	60.0	50.0	0.0	90.0
2	57.1	42.9	85.7	71.4	100.0	71.4	42.9	85.7	0.0	28.6
3	50.0	30.0	100.0	30.0	100.0	60.0	70.0	90.0	0.0	40.0
4	37.5	62.5	25.0	12.5	100.0	7.5	25.0	62.5	0.0	30.0
5	66.7	41.7	25.0	0.0	100.0	16.7	83.3	41.7	0.0	58.3
6	57.1	14.3	100.0	85.7	42.9	100.0	100.0	0.0	0.0	42.9
7	50.0	75.0	75.0	0.0	25.0	100.0	25.0	75.0	0.0	25.0
8	40.0	100.0	20.0	0.0	60.0	60.0	20.0	60.0	20.0	50.0
9	42.9	14.3	57.1	14.3	100.0	57.1	28.6	28.6	0.0	28.6
10	40.0	0.0	80.0	100.0	100.0	100.0	60.0	80.0	20.0	20.0
<hr/>										
\bar{R}	51.13	43.07	65.78	35.39	77.79	67.27	51.48	57.35	4.00	41.34
s	11.42	30.49	31.82	37.68	29.91	34.39	27.58	28.17	8.43	20.84

NOTE: The second decimal place in \bar{R} and s is not significant.

APPENDIX D
Statistical Analysis

APPENDIX D

Analysis of Variance

(test for significance of size and tone reproduction)

	Size			
	Ⓢ	Ⓜ	Ⓛ	
1	55.00	51.61	51.13	157.74
2	19.79	60.71	43.07	123.57
3	56.22	80.08	65.78	202.08
4	46.84	35.65	35.39	117.93
TR 5	76.83	67.20	77.79	221.82
6	62.47	71.77	67.27	201.51
7	70.47	74.93	51.48	196.88
8	45.81	71.92	57.35	175.08
9	.00	10.33	4.00	14.33
10	72.70	61.15	41.34	175.19
	506.18	585.35	494.60	1586.13

$$\gamma = 29 \quad SST = \sum x_{ij}^2 - \frac{T_{..}^2}{n} = 97553.44 - \frac{2515805.37}{30} = 13693.17$$

$$\gamma = 9 \quad SSR = \frac{T_i^2}{c} - \frac{T_{..}^2}{n} = \frac{285017.23}{3} - \frac{2515808.37}{30} = 11145.47$$

$$\gamma = 2 \quad SSC = T_{.j}^2 - \frac{T_{..}^2}{n} = \frac{843481.97}{10} - 83860.27 = 487.92$$

$$\gamma = 18 \quad SSE = SST - (SSR + SSC) = 2059.78$$

	MS	F ratio	Limit	($\alpha = .05, \beta = .05$)
SSR	1238.38	10.82	(2.46)	SIGNIFICANT
SSC	243.96	2.13	(3.55)	NOT SIGNIFICANT
SSE	114.43			

APPENDIX D

Analysis of Variance

(test for significance of sharpness and tone reproduction)

	Sharpness		
	(S)	(U)	
1	51.61	54.61	106.22
2	60.71	51.34	112.05
3	80.08	63.94	144.02
4	35.65	37.25	72.90
<u>TR</u> 5	67.20	70.83	138.03
6	71.77	62.47	134.24
7	74.93	77.23	152.16
8	71.92	46.46	118.38
9	10.33	4.50	14.83
10	61.15	55.89	117.04
	585.35	524.52	1109.87

$$\gamma = 19 \quad SST = 69613.01 - 61590.57 = 8022.44$$

$$\gamma = 9 \quad SSR = \frac{138051.46}{2} - 61590.57 = 7435.16$$

$$\gamma = 1 \quad SSC = \frac{617755.85}{10} - 61590.57 = 185.01$$

$$\gamma = 9 \quad SSE = 402.27$$

 $(\alpha = .05, \beta = .05)$

	MS	F ratio	Limit	
SSR	826.12	18.48	(3.18)	SIGNIFICANT
SSC	185.01	4.13	(5.12)	NOT SIGNIFICANT
SSE	44.69			

APPENDIX E

Least Squares Curve Fitting

APPENDIX E

Least Squares Analysis

A. Procedure

With the final sets of prints evaluated and analysed, each print is characterized by its size, tone reproduction, and quality rating (\bar{R}). The least squares method allows one to find a relationship between quality and tone reproduction at a given size. If a relationship of the form

$$Q = a_1 \lambda_1 + a_2 \lambda_2 + a_3 \lambda_1^2 + a_4 \lambda_2^2 + a_5 \lambda_1 \lambda_2 + a_6$$

is assumed, then the coefficients (a_i) may be calculated which describe the curve best fitting the original data points (i.e. producing the lowest squared deviation from the original data).

The procedure used to find the coefficients is to solve simultaneously a set of "normal equations" formed by multiplying through the original equation by each of the independent variables and summing over the n sets of data to be fitted to the curve.

These normal equations are of the form:

$$(1) \quad \Sigma Q \lambda_1 = a_1 \Sigma \lambda_1^2 + a_2 \Sigma \lambda_1 \lambda_2 + a_3 \Sigma \lambda_1^3 + a_4 \Sigma \lambda_1 \lambda_2^2 + a_5 \Sigma \lambda_1^2 \lambda_2 + a_6 \Sigma \lambda_1$$

....

$$(5) \quad \Sigma Q \lambda_1 \lambda_2 = a_1 \Sigma \lambda_1^2 \lambda_2 + a_2 \Sigma \lambda_1 \lambda_2^2 + a_3 \Sigma \lambda_1^3 \lambda_2 + a_4 \Sigma \lambda_1 \lambda_2^3 + a_5 \Sigma \lambda_1^2 \lambda_2^2 + a_6 \Sigma \lambda_1 \lambda_2$$

$$(6) \quad \Sigma Q = a_1 \Sigma \lambda_1 + a_2 \Sigma \lambda_2 + a_3 \Sigma \lambda_1^2 + a_4 \Sigma \lambda_2^2 + a_5 \Sigma \lambda_1 \lambda_2 + a_6 n$$

When the coefficients are determined the quality points (\bar{R}_c) corresponding to any tone reproduction may be calculated.

To find the tone reproduction which will produce a maximum value of Q , the first and second derivatives of the equation must be investigated.

For the original equation

$$Q = a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_1^2 + a_4\lambda_2^2 + a_5\lambda_1\lambda_2 + a_6$$

two partial first derivatives may be obtained which when set equal to zero and solved simultaneously will yield values of λ_1 and λ_2 which may produce a maximum value of Q .

These derivatives are:

$$\frac{\partial Q}{\partial \lambda_1} = a_1 + 2a_3\lambda_1 + a_5\lambda_2 = 0$$

$$\frac{\partial Q}{\partial \lambda_2} = a_2 + a_5\lambda_1 + 2a_4\lambda_2 = 0 \quad .$$

The final check is made by using the second derivatives:

$$\frac{\partial^2 Q}{\partial \lambda_1^2} = 2a_3 \quad \text{and} \quad \frac{\partial^2 Q}{\partial \lambda_2^2} = 2a_4 \quad .$$

If these two values are negative, the values of λ_1 and λ_2 will produce a maximum Q .

B. Computer Program to Fit Data to a Curve of the Form

$$Q = a_1 \lambda_1 + a_2 \lambda_2 + a_3 \lambda_1^2 + a_4 \lambda_2^2 + a_5 \lambda_1 \lambda_2 + a_6$$

```

1.000      DIMENSION D(20),Z(36),W(6),X(10),Y(10),Q(10),QQ(10)
2.000      DIMENSION ZZ(16),WW(4)
3.000      DO 3I=1,20
4.000      D(I)=0.0
5.000      3 CONTINUE
6.000      DO 4 I=1,10
7.000      READ 9,X(I),Y(I),Q(I)
8.000      5 D(1)=X(I)+D(1)
9.000      D(2)=Y(I)+D(2)
10.000     D(3)=X(I)*X(I)+D(3)
11.000     D(4)=Y(I)*Y(I)+D(4)
12.000     D(5)=X(I)*Y(I)+D(5)
13.000     D(6)=X(I)*X(I)*X(I)+D(6)
14.000     D(7)=Y(I)*Y(I)*Y(I)+D(7)
15.000     D(8)=X(I)*(Y(I)*Y(I))+D(8)
16.000     D(9)=Y(I)*(X(I)*X(I))+D(9)
17.000     D(10)=(X(I)*X(I))*(X(I)*X(I))+D(10)
18.000     D(11)=(Y(I)*Y(I))*Y(I)*Y(I)+D(11)
19.000     D(12)=(X(I)*X(I))*(Y(I)*Y(I))+D(12)
20.000     D(13)=(X(I)*X(I))*X(I)*Y(I)+D(13)
21.000     D(14)=(Y(I)*Y(I))*Y(I)*X(I)+D(14)
22.000     D(15)=Q(I)*X(I)+D(15)
23.000     D(16)=Q(I)*Y(I)+D(16)
24.000     D(17)=Q(I)*(X(I)*X(I))+D(17)
25.000     D(18)=Q(I)*(Y(I)*Y(I))+D(18)
26.000     D(19)=Q(I)*(X(I)*Y(I))+D(19)
27.000     D(20)=Q(I)+D(20)
28.000     4 CONTINUE
29.000     8 PRINT 24,(D(I),I=1,20)
30.000     9 FORMAT(F6.4)
31.000     10 FORMAT(F10.5)
32.000     Z(1)=D(3)
33.000     Z(2)=D(5)
34.000     Z(3)=D(6)
35.000     Z(4)=D(8)
36.000     Z(5)=D(9)
37.000     Z(6)=D(1)
38.000     Z(7)=D(5)
39.000     Z(8)=D(4)
40.000     Z(9)=D(9)
41.000     Z(10)=D(7)
42.000     Z(11)=D(8)
43.000     Z(12)=D(2)
44.000     Z(13)=D(6)
45.000     Z(14)=D(9)
46.000     Z(15)=D(10)
47.000     Z(16)=D(12)
48.000     Z(17)=D(13)
49.000     Z(18)=D(3)
50.000     Z(19)=D(8)

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52.0000      Z(21)=D(12)
53.0000      Z(22)=D(11)
54.0000      Z(23)=D(14)
55.0000      Z(24)=D(4)
56.0000      Z(25)=D(9)
57.0000      Z(26)=D(8)
58.0000      Z(27)=D(13)
59.0000      Z(28)=D(14)
60.0000      Z(29)=D(12)
61.0000      Z(30)=D(5)
62.0000      Z(31)=D(1)
63.0000      Z(32)=D(2)
64.0000      Z(33)=D(3)
65.0000      Z(34)=D(4)
66.0000      Z(35)=D(5)
67.0000      Z(36)=10.0
68.0000      W(1)=D(15)
69.0000      W(2)=D(16)
70.0000      W(3)=D(17)
71.0000      W(4)=D(18)
72.0000      W(5)=D(19)
73.0000      W(6)=D(20)
74.0000      ZZ(1)=D(3)
75.0000      ZZ(2)=D(5)
76.0000      ZZ(3)=D(6)
77.0000      ZZ(4)=D(8)
78.0000      ZZ(5)=D(5)
79.0000      ZZ(6)=D(4)
80.0000      ZZ(7)=D(9)
81.0000      ZZ(8)=D(7)
82.0000      ZZ(9)=D(6)
83.0000      ZZ(10)=D(9)
84.0000      ZZ(11)=D(10)
85.0000      ZZ(12)=D(12)
86.0000      ZZ(13)=D(8)
87.0000      ZZ(14)=D(7)
88.0000      ZZ(15)=D(12)
89.0000      ZZ(16)=D(11)
90.0000      WW(1)=D(15)
91.0000      WW(2)=D(16)
92.0000      WW(3)=D(17)
93.0000      WW(4)=D(18)
94.0000      CALL SIMULT(Z,W,6,J)
95.0000      PRINT 11
96.0000      PRINT 10,(W(I),I=1,6)
97.0000      11 FORMAT(/
98.0000          WW1=W(1)*D(3)+W(2)*D(5)+W(3)*D(6)+W(4)*D(8)+
99.0000          1W(5)*D(9)+W(6)*D(1)-D(15)
100.0000      WW2=W(1)*D(5)+W(2)*D(4)+W(3)*D(9)+W(4)*D(7)+
101.0000      1W(5)*D(8)+W(6)*D(2)-D(16)
102.0000      WW3=W(1)*D(6)+W(2)*D(9)+W(3)*D(10)+W(4)*D(12)+
103.0000      1W(5)*D(13)+W(6)*D(3)-D(17)
104.0000      WW4=W(1)*D(8)+W(2)*D(7)+W(3)*D(12)+W(4)*D(11)+
105.0000      1W(5)*D(14)+W(6)*D(4)-D(18)
106.0000      WW5=W(1)*D(9)+W(2)*D(8)+W(3)*D(13)+W(4)*D(14)+
107.0000      1W(5)*D(12)+W(6)*D(5)-D(19)
108.0000      WW6=W(1)*D(1)+W(2)*D(2)+W(3)*D(3)+W(4)*D(4)+
109.0000      1W(5)*D(5)+W(6)*10.0-D(20)
110.0000      WW7=WW(1)*D(3)+WW(2)*D(5)+WW(3)*D(6)+WW(4)*D(8)-D(15)
111.0000      WW8=WW(1)*D(5)+WW(2)*D(4)+WW(3)*D(9)+WW(4)*D(7)-D(16)
112.0000      WW9=WW(1)*D(6)+WW(2)*D(9)+WW(3)*D(10)+WW(4)*D(12)-D(17)
113.0000      WW10=WW(1)*D(8)+WW(2)*D(7)+WW(3)*D(12)+WW(4)*D(11)-D(18)

```



```

114.000      PRINT 11
115.000      PRINT 25,WW1,WW2,WW3,WW4,WW5,WW7
116.000 C    PRINT 26,WW7,WW8,WW9,WW10
117.000      SUM=0.0
118.000      DO 20 I=1,10
119.000      QQ(I)=W(1)*X(I)+W(2)*Y(I)+W(3)*X(I)**2.0+W(4)*Y(I)**2.0
120.000      1+W(5)*X(I)*Y(I)+W(6)
121.000 C    QQ(I)=WW(1)*X(I)+WW(2)*Y(I)+WW(3)*X(I)*X(I)+WW(4)*Y(I)*Y
( I )
122.000      SUM=QQ(I)-Q(I)+SUM
123.000      20 CONTINUE
124.000      PRINT 11
125.000      PRINT 30,((X(I),Y(I),Q(I),QQ(I)),I=1,10)
126.000      PRINT 11
127.000      PRINT 27,SUM
128.000      PRINT 11
129.000      PRINT 11
130.000      24 FORMAT(5F12.5)
131.000      25 FORMAT(6F10.3)
132.000      26 FORMAT(4F10.3)
133.000      30 FORMAT(5X,F10.5,5X,F10.5,8X,F4.2,8X,F4.2)
134.000      27 FORMAT(19HSUM OF DIFFERENCES=,F5.3)
135.000      END
--EOF HIT AFTER 135.
*END

```

SEV.LEV. = 0

XEQ7 Y

P	.00000	.00000	1.00068	.99888	.00015
	.02107	.30293	-.10452	.00577	.17312
	.30418	.07325	.00959	-.05412	14.84820
	-44.20616	47.63129	33.94461	8.81142	528.19946

11.35617	} a_i
-21.80235	
-99.85713	
-49.94768	
64.32507	
67.80072	

	-.000	.000	.000	.000	-.000	-.000
	λ_1		λ_2	\bar{R}	\bar{R}_C	
P	-.09600	.02800	53.10	64.97		
	-.49300	-.02400	43.70	39.19		
	.09700	-.14800	66.50	69.17		
	.50900	.01300	38.80	47.84		
	-.22500	-.14000	73.20	64.29		
	.37200	-.11300	66.00	57.33		
	.16000	-.35400	68.50	64.88		
	-.33600	-.38800	55.90	62.04		
	-.27000	.69200	4.70	6.43		
	.28200	.43400	57.80	52.06		

SUM OF DIFFERENCES= -.001

Small Reproduction

34.62999	} a_i
-10.95260	
-139.40141	
-41.12891	
109.40222	
68.67635	

- .000	.000	.000	.000	.000 \bar{R}_C	.000
λ_1		λ_2	\bar{R}		
-.09600		.02800	55.00	63.43	
-.49300		-.02400	19.80	19.26	
.09700		-.14800	56.20	69.87	
.50900		.01300	46.90	50.76	
-.22500		-.14000	76.80	58.00	
.37200	P	-.11300	62.50	58.38	
.16000		-.35400	70.50	63.18	
-.33600		-.38800	45.80	53.62	
-.27000		.69200	.00	1.45	
.28200		.43400	72.70	68.25	

SUM OF DIFFERENCES=-.000

Large Reproduction

-.57449	} a_i
-20.90234	
-86.28905	
-71.40926	
39.99174	
65.23712	

.000	.000	.000	.000	-.000	.000
λ_1		λ_2	\bar{R}	\bar{R}_C	
-.09600		.02800	51.10	63.75	
-.49300		-.02400	43.10	45.48	
.09700		-.14800	65.80	65.32	
.50900		.01300	35.40	42.57	
-.22500		-.14000	77.80	63.78	
.37200		-.11300	67.30	52.85	
.16000		-.35400	51.50	59.12	
-.33600		-.38800	57.40	58.26	
-.27000		.69200	4.00	2.97	
.28200		.43400	41.30	40.59	

SUM OF DIFFERENCES=-.001

Medium Reproduction

$$\left. \begin{array}{r} 3.42284 \\ -33.97911 \\ -77.02173 \\ -25.78845 \\ 71.29605 \\ 68.81232 \end{array} \right\} a_i$$

$-.000$	$.000$	$.000$	$.000$	$-.000$	$.000$
λ_1		λ_2	\bar{R}	\bar{R}_C	
$-.09600$		$.02800$	51.60	66.61	
$-.49300$		$-.02400$	60.70	50.05	
$.09700$		$-.14800$	80.10	71.86	
$.50900$		$.01300$	35.60	50.63	
$-.22500$		$-.14000$	67.20	70.64	
$.37200$		$-.11300$	71.80	59.94	
$.16000$		$-.35400$	74.90	72.15	
$-.33600$		$-.38800$	71.90	77.56	
$-.27000$		$.69200$	10.30	13.09	
$.28200$		$.43400$	61.20	52.77	

SUM OF DIFFERENCES=-.001

Medium Unsharp Reproduction

$$\left. \begin{array}{r} 8.27077 \\ -20.05353 \\ -97.76053 \\ -65.05188 \\ 32.01984 \\ 68.72015 \end{array} \right\} a_i$$

$-.000$	$.000$	$.000$	$.000$	$-.000$	$.000$
λ_1		λ_2	\bar{R}	\bar{R}_C	
$-.09600$		$.02800$	54.60	66.33	
$-.49300$		$-.02400$	51.30	41.70	
$.09700$		$-.14800$	63.90	69.69	
$.50900$		$.01300$	37.20	47.54	
$-.22500$		$-.14000$	70.80	64.45	
$.37200$		$-.11300$	62.50	58.36	
$.16000$		$-.35400$	77.20	64.67	
$-.33600$		$-.38800$	46.50	57.07	
$-.27000$		$.69200$	4.50	8.35	
$.28200$		$.43400$	55.90	46.24	

SUM OF DIFFERENCES=-.000